HIGH SCHOOL MATH

The student will

use the hierarchy of number to classify numbers.

determine the degree and leading coefficient of a polynomial.

evaluate number sentences and expressions.

use the order of operations to evaluate mathematical expressions.

add, subtract, multiply and divide polynomials.

find absolute values.

add, subtract, multiply and divide imaginary and complex numbers.

find the real roots of real numbers.

simplify numbers with positive and negative exponents.

simplify expressions involving radicals.

convert between radians and degrees.

evaluate factorials.

determine the dimensions of matrices and perform the operations of addition, subtraction, and multiplication.

multiply and divide monomials with positive and negative exponents.

evaluate logarithms, both common and natural.

calculate the distance between two points.

find the sine, cosine, and tangent of an angle.

use trigonometry to fine the missing sides or angles of a right triangle.

name the exact values of the sine, cosine, and tangent of the special triangles. (30-60-90 right triangle
  and isosceles right triangle).

use the unit circle to find the sine, cosine, and tangent of the special angles in quadrants II, III, and IV.

use the Law of Cosines and the Law of Sines to solve problems.

use the concept of angle bisector to find angle measures.

apply the properties of complementary and supplementary angles to solve for angle measures. define a function and
  identify the dependent and the independent variable.

find the range and the domain of a function.

create a formula for a function.

determine if a relation is a function.

compose functions.

solve direct and inverse variations.

graph exponential and logarithmic functions.

expand binomials. (FOIL)

write polynomials as the product of factors.

find the slope of a line given two points on a line or an equation for a line.

graph linear, quadratic, logarithmic and exponential functions.

graph polynomial functions or 3rd degree or higher.

find the asymptotes for a given relation or function.

find an equation for a line parallel or perpendicular to a given line.

graph a line using the slope and the y-intercept method.

graph a line using the x- and y- intercepts.

write an equation for a line in slope/y-intercept form and in standard form.

find the equation for a line given two points or given a point and the slope.

use matrices to perform transformations. (translations, reflections, rotations, size change) determine if two lines
  are parallel, perpendicular or neither.

solve a system of equations or inequalities by graphing.

solve and graph an absolute value.

graph the image of a function using the Graph-Translation theorem.

find the center and the length of the radius of a circle and graph it.

graph the sine and cosine functions.

use the formulas for perimeter, area, and volume to solve problems.

use the Pythagorean Theorem to solve problems involving right triangles.

determine whether a parabola opens up or down, find its vertex, and determine whether it is a maximum
  or a minimum value.

complete direct proof arguments.

write and solve proportions.

write and solve an algebraic sentence involving a real life situation.

write and solve equations involving direct and inverse variation.

fit a mathematical model (linear or quadratic) to real life data.

create and solve systems of equations for real life situations.

calculate compound interest.

solve and graph problems involving geometric growth and decay.

find the measures of central tendency.

solve problems using probability.

solve problems using combinations and permutations.

solve equations for a given variable.

write the explicit and recursive formulas for a sequence.

find the sum of a sequence.

find the point of intersection between two lines.

determine from an equation whether a line is horizontal, vertical, or neither.

solve a system of equations algebraically.

solve and graph quadratic equations.

complete the square of a binomial.

write the equation for a parabola in standard and vertex form.

use the Quadratic Formula to solve a quadratic equation.

use the discriminant test to determine the number of real roots for a quadratic equation. apply the Factor
  Theorem to find the zeros of a polynomial function.

use the Rational Root Theorem to find the rational roots of a polynomial.

solve equations involving positive and negative exponents and using radicals.

rationalize the denominator of a fraction with a radical in the denominator.

use logarithms to solve exponential equations.

solve equations using the trigonometry ratios.

write the equation for a polynomial as the product of its factors.

solve equations involving logarithms.

find the vertices, the foci, and the equations for the asymptotes of a hyperbola and graph this figure.

find the vertices and foci of an ellipse and graph this figure.

write a polynomial equation given the zeros

solve a rational equation

graph points on a scatter plot and use technology to find the line of best fit

graph a rational function

add or subtract rational expressions

multiply or divide rational expressions

Instructions for the User

- To use <u>Summer Math Skills Sharpener,</u> simply tear off a page and complete it. The program is designed to be used 3 days per week for 12 weeks. Please allow about an hour per page.

- Detailed solutions to all problems are included at the back of the book. Please complete an entire sheet prior to checking your answers.

- All concepts are part of standard geometry and advanced algebra curriculums. Please attempt all problems. In addition to the solutions, pink "Help Pages" have been included to assist you in completing the problems.

- A yellow "Glossary of Terms" is located at the back of the book.

- Pages should be worked in order. While each page contains mixed concepts, individual concepts within the book have been ordered from easier to more difficult.

- If you experience difficulty with certain concepts, address the problems with your teacher in the fall. He or she may recommend additional help in these areas.

- It is important to give every problem your best effort. Problems may seem challenging, but use a combination of the "Help Pages" and the "Solutions" to assist you for maximum success.

- This book can be used to supplement other placement test reviews. Among the problems are concepts that appear regularly on these tests.

For problems 1 - 4, simplify.

1. $\sqrt[3]{125}$ 　　　 2. $\dfrac{1}{\sqrt{2}}$ 　　　 3. $\sqrt{27} + \sqrt{75}$ 　　　 4. $\sqrt{1+8}$

5. Given: $\overline{AB}$ is shown at the right.

   $\xleftarrow{\hspace{1cm}} \overset{A}{\underset{-2}{\rule{0pt}{1pt}}} \rule{4cm}{0.5pt} \overset{B}{\underset{6}{\rule{0pt}{1pt}}} \xrightarrow{\hspace{1cm}}$

   a. Find the midpoint of $\overline{AB}$.

   b. Find the distance from the midpoint to each endpoint.

   c. Write an absolute value inequality to represent the points on the graph of $\overline{AB}$.

6. Three fair coins are tossed. What is the probability that exactly two will be tails?

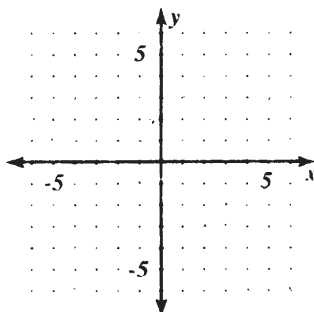7. In parallelogram $ABCD$ $m\angle A = 2x - 22$ and $m\angle D = x - 1$.

   a. Find x.

   b. Find the measure of each angle of the parallelogram.
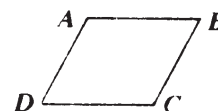
8. Give the image of $(3,-6)$ translated by vector $(4,3)$.

9. Give an equation for a line with slope $= 3$ and containing point $(1,3)$.

10. Graph $x + 3y = 9$.

11. Jeff works each summer to earn money for college and recreation. Each month he divides his salary in the following way: $\dfrac{1}{2}$ goes into savings, $\dfrac{1}{5}$ is for his car, $\dfrac{1}{4}$ is spent on clothing and the rest is used on recreational activities. Last month he spent \$25 on recreation.

   How much did he earn?

For problems 1 - 4, simplify.

1. $\sqrt{-72}$ 2. $\sqrt{-3} \cdot \sqrt{-6}$ 3. $\dfrac{\sqrt{-20}}{\sqrt{-5}}$ 4. $\dfrac{1}{\sqrt{-16}}$

5. The sixth term of a geometric sequence is 98304. The constant ratio is 4.

     a. Find the first term of the ratio.

     b. Give the formula for the nth term.

6. If $e^{2x} = 10$ find x. (Round to the nearest hundredth.)

7. If Θ is an acute angle and the $\sin \Theta = \dfrac{4}{5}$, find:

     a. $\cos\Theta$ b. $\tan\Theta$

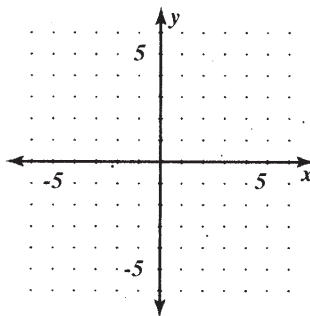8. A point on the terminal side of an angle with measure Θ is (-7,2). Find:

     a. $\sin\Theta$ b. $\cos\Theta$ c. $\tan\Theta$

9. In a certain class 55% are girls and 35% are in the band. Given that a girl has been chosen, what is the probability she is in the band?

10. Graph: $\begin{cases} y \leq -2x + 1 \\ y \geq x - 4 \end{cases}$

11. Use a graphing utility to find the zeros
of $y = -x^4 + 3x^3 + x^2 - 3x$

12. The area of a rectangular garden is 170 square feet. The length of the garden is 3 feet less than twice the width. Find the dimensions of the garden.

13. The ratio of the angles of a right triangle is $2:\sqrt{3}:1$. Find the sine and cosine ratios for the two acute angles.

For problems 1 – 3, simplify.

1. $\dfrac{3}{x} + \dfrac{6}{x+2}$

2. $\dfrac{x-2}{x^2-1} - \dfrac{x}{x-1}$

3. $\dfrac{5x}{x^2-4} + \dfrac{x}{x^2-4x+4}$

For 4 and 5 determine the values of x for which the function is not defined.
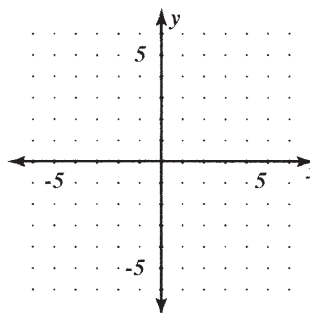
4. $h(x) = \dfrac{x^2}{x^2-1}$

5. $f(x) = \dfrac{x-2}{x^2-2x-15}$

6. On the same coordinates, graph:

   a. $y = 3^x$

   b. $y = \log_3 x$

   c. Find the symmetry line for the two graphs.

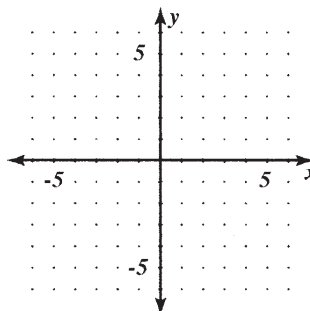7. Find the zeros of: $f(x) = x^3 + x^2 - 5x + 3$

8. Consider the hyperbola $\dfrac{y^2}{25} - \dfrac{x^2}{9} = 1$

   a. Graph the relation.

   b. Label the vertices.

   c. Find and label the foci.

   d. Find and label the asymptotes.

9. A certain algae population doubles every two days. Its initial size is 1000 organisms. How many will be present after:

   a. 10 days?

   b. 90 days?

10. The ACT consists of 60 multiple choice questions, each with four responses. If you have to guess on eight of them, what is the probability you will answer them all correctly?

acute angle: An angle measuring greater than $0°$ and less than $90°$.

adjacent angles: Two nonstraight and nonzero angles with a common side interior to the angles formed by the uncommon sides.

algebraic equation: A math sentence relating two expressions as equal.

altitude: The perpendicular distance from the vertex of a triangle to the side opposite. Also, the perpendicular distance between parallel lines.

angle: The union of two rays (the sides) at a point (the vertex).

angle bisector: A ray that divides an angle into two equal parts.

angle of depression: The angle measured from the horizontal downward from the observer's eye.

angle of elevation: The angle measured from the horizontal upward from an observer's eye.

area: The number of unit squares or parts of unit squares required to tile a plane figure.
ex.
parallelogram: $A = hb$
rectangle: $A = lw$
triangle: $A = \dfrac{1}{2}hb$
trapezoid: $A = \dfrac{1}{2}h(b_1 + b_2)$
circle: $A = \pi r^2$

arithmetic sequence: A sequence with a constant difference.

base: The variable b in the expression b^n.

binomial: A polynomial containing two terms.

bisector: A point, line, ray, or plane that divides a segment, angle, or figure into two parts of equal measure.

circle: The set of all points (the radius), equal distance from a certain point (the center).

circumference: The perimeter of a circle. $c = \pi d$. The ratio of the circumference to the diameter is π

collinear points: Points that lie on the same line.

complementary angles: Two angles whose sum is $90°$.

complex number: A number that can be written in the form $a + bi$.

composite of functions: An operation that first applies one function, then the other. It is written $f(g(x))$ or $f \circ g(x)$. Both mean f following g of x.

cone: The surface of a conic solid whose base is a circle.

conic solid: The set of points between a given point (the vertex) and the points in a given region (the base), and the vertex and the base.

coordinates: An ordered pair.

PROPERTIES OF POWERS:

Zero power Property: $b^0 = 1$

Power identity: $b^1 = 1b$

Product of Powers Property: $b^m \cdot b^n = b^{m+n}$

Power of a Power Property: $\left(b^n\right)^m = b^{nm}$

Power of Product Property: $(ab)^n = a^n b^n$

Quotient of Powers Property: $\dfrac{b^n}{b^m} = b^{n-m}$

Power of a Quotient Property: $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

PROPERTIES OF LOGARITHMS:

Definition of a logarithm: If $\log_b x = y$ then $b^y = x$. Notice that a logarithm is an exponent. As you learn the properties of logarithms relate them to the more familiar properties of powers.

Logarithm of 1: $\log_b 1 = 0 \quad \{b^0 = 1\}$

Logarithm of b to the base b: $\log_b b = 1 \quad \{b^1 = b\}$

Logarithm of b^n to the base b: $\log_b b^n = n \quad \{b^n = b^n\}$

Product Property of logarithms: $\log_b(mn) = \log_b m + \log_b n$

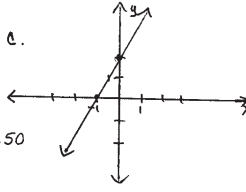**Quotient Property of logarithms:** $\log_b\left(\dfrac{m}{n}\right) = \log_b m - \log_b n$

Power Property of logarithms: $\log_b\left(m^n\right) = n\log_b m$

SOLUTIONS

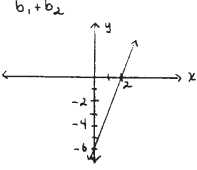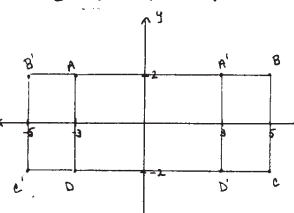## pg. 1

① $3x + 9 = -15$ $3(-8) + 9 = -15$
   $3x = -24$ $-15 = -15 ✓$
    $x = -8$

② $\frac{x+3}{2} = \frac{3}{4}$ $\frac{-1.5+3}{2} = \frac{3}{4}$
   $6 = 4x + 12$ $\frac{1.5}{2} = \frac{3}{4}$
   $-6 = 4x$
    $x = -1.5$ $\frac{3}{4} = \frac{3}{4} ✓$

③ $6\left(\frac{1}{3}x + \frac{1}{2}x + 80\right) = (75)6$ $\frac{1}{3}(-6) + \frac{1}{2}(-6) + 80 = 75$
   $2x + 3x + 480 = 450$ $-2 + -3 + 80 = 75$
     $5x = -30$ $75 = 75 ✓$
     $x = -6$

④ $C = 1.09(119.95)x + 5$

⑤ a) $d = 5$ $a_5 = 24$, $a_6 = 29$
   b) $a_n = a_1 + d(n-1)$
    $a_n = 4 + 5(n-1)$
    $a_n = 4 + 5n - 5 = 5n - 1$

⑥ $6x - 31 = 2x + 47$ $m\angle 1 = m\angle 3 = 86$
   $4x = 78$ $m\angle 2 = m\angle 4 = 94$
    $x = 19.5$
  $6(19.5) - 31 = 86$

⑦ $y = (x+5)^2 + 8$

⑧ $(x+y)(a+3) =$
  $ax + 3x + ay + 3y$

⑨ Let x = original test sum
   $\frac{x}{4} = 83$ $x = 332$
   $332 - 80 + 90 = 342$
     $\frac{342}{4} = 85.5$

⑩ $4 = 2^2$ $5 = 5 \cdot 1$ $6 = 2 \cdot 3$
   $2^2 \cdot 5 \cdot 3 = 60$
   $60 + 1 = 61$ pieces of candy

⑪ $\frac{9-1}{16-1} = \frac{8}{15}$

pg. 2

① $3(2x+1) + (x+1) = 11$
   $6x + 3 + x + 1 = 11$
    $7x = 7$
    $x = 1$
  $3(3) + 2 = 11$
   $11 = 11 ✓$

② $.06x + .045(1000 - x) = 93$
   $.06x + 45 - .045x = 93$
    $.015x = 48$
    $x = 3200$
  $192 + -99 = 93$
   $93 = 93 ✓$

③ $7x - 6(11 - 2x) = 10$
  $7x - 66x + 12x = 10$
   $19x = 76$
   $\boxed{x = 4}$

④ $4y - \frac{1}{10} = 3y + \frac{4}{5}$
    $y = \frac{4}{5} + \frac{1}{10} =$
    $\boxed{y = \frac{9}{10}}$

⑤ a. $4x - 2y = -4$ $(-1,0), (0,2)$
  b. $-2y = -4x - 4$ $m = 2$
    $y = 2x + 2$
  c.

⑥ $p = kd^2$ $p = .0995 \cdot 14^2$ #19.50
  $9.95 = k \cdot 10^2$ $p = 19.502$
  $.0995 = k$

⑦ $x + 52 - 7x - 16 = 180$ $-24 + 52 = 28°$
   $-6x + 36 = 180$ $-7(-24) - 16 = 152°$
    $-6x = 144$
    $x = -24$

⑧ $A' = (-2,-2)$ $B' = (3,-7)$ $C' = (0,4)$

⑨ a) $\frac{3}{5} = \frac{x}{450}$ b) $5x = 1350$
     $x = 270$ girls

⑩ $x^2 < 16$ $x < 4$ or $x > -4$
    $-4 < x < 4$

⑪ a. $d = 8$ arithmetic $6, 14, 22, 30$
  b. $a_n = 6 + 8(n-1)$
  c. $\begin{cases} a_1 = 6 \\ a_n = a_{n-1} + 8 \ n > 1 \end{cases}$
  d. $a_{20} = 6 + 8(19) = 158$

pg. 3

① $A = bh$ ② $A = \frac{1}{2}h(b_1 + b_2)$ ③ $V = \pi r^2 h$
  $h = \frac{A}{b}$ $\frac{2A}{b_1 + b_2} = h$ $h = \frac{V}{\pi r^2}$

④ a) 2
   b) -6

⑤ Let x = smaller
   $3x + 4$ = larger
   $x + 3x + 4 = 90$
    $4x = 86$
    $x = 21.5$
  $3(21.5) + 4 = 68.5$

⑥ $x^2 - \pi\left(\frac{x}{2}\right)^2 =$
  $x^2 - \pi \cdot \frac{x^2}{4} \approx .215 x^2$

⑦ Total $= 17$
  $\frac{4}{17} \cdot \frac{3}{16} = \frac{12}{272} = \frac{3}{68}$

⑧ $0; 1; 2$

⑨ a) $A' = (3,2)$ $B' = (-5,2)$
    $C' = (-5,-2)$ $D' = (3,-2)$

⑩ a) mean = $33450
    median = $23000
  b) The median is closer
   to the majority of
   the wages.

⑪ $65 = a_1 + 4(18)$
   $a_1 = -7$

pg. 4

① $x^2 + 3x - 10$ ② $x^2 - 16x + 64$ ④ $x^2 + 2x + 1$

③ $x^2 - 25$ $\dfrac{x+1}{x^3 + 2x^2 + x}$
          $\dfrac{x^2 + 2x + 1}{x^3 + 3x^2 + 3x + 1}$

⑤ $5 \leq x + 8$
   $-3 \leq x$

⑥ $\begin{cases} l = 3r \\ r + l \leq 30 \end{cases}$

⑦ $I = \dfrac{k}{d^2}$ $I = \dfrac{6300}{10^2}$
  $700 = \dfrac{k}{3^2}$ $I = 63$ units
   $k = 6300$

⑧ a. Let c = Milo's books
   $\frac{1}{5}c + 18 = 24$
  b. $\frac{1}{5}c = 6$
   $5 \cdot \frac{1}{5}c = 6 \cdot 5$
    $c = 30$

⑨ a. $m = \frac{4-2}{6-(-5)} = \frac{2}{11}$
  b. $m_{||} = \frac{2}{11}$ c. $m_\perp = \frac{-11}{2}$

⑩ a) $4x + 7 = 6x - 11$ b) $4(9) + 7 = 43$
   $18 = 2x$ $m\angle D = m\angle C = 43°$
   $x = 9$ $m\angle A = m\angle B = 137°$

⑪ 680
  $2 \diagup 340$
    $2 \diagup 170$
     $2 \diagup 85$
      $5 \diagup 17$

$2^3 \cdot 5 \cdot 17$

⑫ Let n = total marbles
   $\frac{3}{n} = \frac{1}{4}$
   $n = 12$
   $12 - 3 = 9$ green