GEOMETRY CURRICULUM OVERVIEW

The student will

- Understand point, line, and plane and their dimensions
- Find the circumference and area of a circle
- Find the perimeter and area of regular geometric figures
- Accurately draw or construct geometric figures using the tools of geometry
- Accurately measure segments and angles
- Solve problems using complementary or supplementary angles
- Distinguish among acute, right, and obtuse angles and triangles
- Demonstrate and understanding of parallel and perpendicular lines
- Graph lines on a number line
- Graph lines in the *xy*-coordinates
- Find the slope and y-intercept of a line
- Use the tools of algebra to solve geometric problems
- Demonstrate an understanding of translations, rotations, and reflections
- Perform size changes and determine their effects on area and volume
- Write a true if-then statement, its converse, its inverse, and its contrapositive
- Demonstrate an understanding of a counterexample
- Show mastery of the common theorems of geometry
- Justify congruence in triangles using the appropriate theorems
- Use CPCFC to find or compare parts of congruent triangles
- Create a logical argument in the form of a proof
- Use proportion to find the missing parts of geometric figures
- Understand and apply the Pythagorean theorem
- Find the surface area and volume of three-dimensional figures
- Decompose a geometric figure to find its area
- Find the sine, cosine, and tangent of an angle
- Show an understanding of the special triangles

- Use the trig functions to find a missing side or angle in a right triangle
- Use the midpoint and distance formulas
- Find an arc measure given a central or inscribed angle
- Find geometric probabilities
- Solve problems involving vectors
- Identify tangent, chord, and secant lines

Instructions for the User

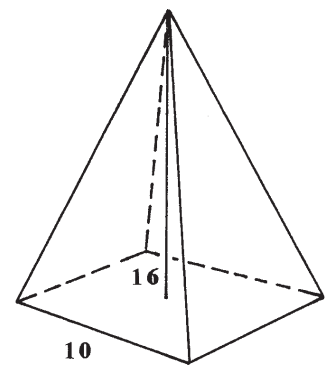
- Summer Math Skills Sharpener, is designed to be used 3 days per week for 12 weeks. Please allow about an hour per page.
- Detailed solutions to all problems are included at the back of the book. Please complete an entire sheet prior to checking your answers.
- All concepts are part of a standard geometry curriculum. Please attempt all problems. In addition to the solutions, pink "Help Pages" have been included to assist you in completing the problems.
- A yellow "Glossary of Terms" is located at the back of the book.
- A green list of "Commonly used Postulates and Theorems" is included to assist you with your proofs.
- Pages should be worked in order. While each page contains mixed concepts, individual concepts have been ordered from easier to more difficult.
- If you experience difficulty with certain concepts, address the problems with your teacher in the fall. He or she may recommend additional help in these areas.
- It is important to give every problem your best effort. Problems may seem challenging, but use a combination of the "Help Pages" and the "Solutions" to assist you for maximum success.
- This book can be used to supplement ACT and other placement test reviews. Among the problems are concepts that appear regularly on these tests.

1. ΔABC is isosceles with base BC. The measure of $\angle A$ is four times the measure of $\angle B$. Find the measure of $\angle C$.

2. B is the midpoint of $\overline{AC}$. If $AB = 3x + 5$ and $BC = 17 - 5x$, find AC.

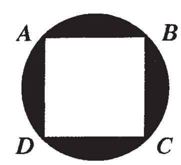
3. Given the regular square pyramid pictured at the right,
   a. find the surface area.

   b. find the volume.

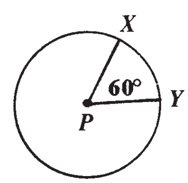


4. One angle of an isosceles triangle measures 42°. What are the possible measures for the other angles of this triangle?

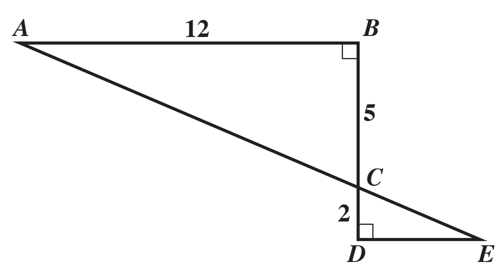
5. Find the area of the shaded region in the figure at the right if $ABCD$ is a square inscribed in circle Θ and $AC = 6\sqrt{2}$.



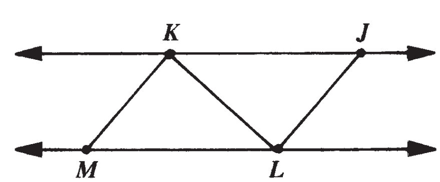
6. ΘP has radius, $r = 5$. Find $m\widehat{XY}$.



7. Given the figure at the right.
   $BD \perp AB$ and $BD \perp DE$. $AB = 12$, $BC = 5$, and $CD = 2$. Find AE.



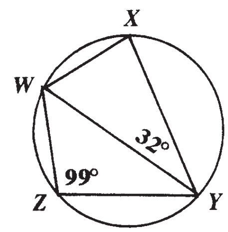
8. Given: $\overleftrightarrow{KJ} \parallel \overleftrightarrow{ML}$, $\overline{KJ} \cong \overline{LM}$
   Prove: $\Delta KJL \cong \Delta LMK$
   (Write your proof on the back of page 16.)



1. **Consider the following statements.**
   a. **Every labrador retriever is a dog.**
   b. **Zeus is a chocolate labrador retriever.**
   What, if anything, can you conclude from these two statements?

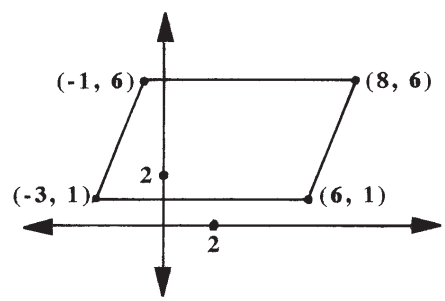
2. **Two similar antique vases are one foot and two feet tall. If it takes one pint of paint to restore the larger vase, how much paint will the smaller vase require?**

3. **Use the circle at the right to find**
   a. $m\overset{\frown}{WXY}$ ____________ b. $m\overset{\frown}{XY}$ ____________
   c. $m\angle WXY$ ____________ d. $m\overset{\frown}{WZY}$ ____________



4. **A parallelogram has vertices (8, 6), (6, 1), (-3, 1), and (-1, 6). Find each of the following:**
   a. **the perimeter of the parallelogram.**

   b. **the area of the parallelogram.**

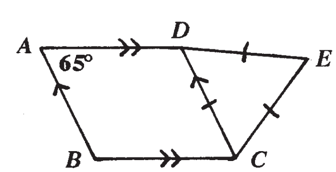


5. **WOOD radio services customers within a 50 mile radius of its broadcasting tower. How large is the area that the station services?**

6. **The angles of a triangle are in a ratio of 1:2:6. Find the measure of each angle.**

7. **A student claims that $\Delta ABC \cong \Delta DEF$ because $AB = DE = 2$, $m\angle A = m\angle D = 35°$, and $m\angle B = m\angle F = 55°$. On the back of page 28, accurately draw and label both triangles to show the error.**

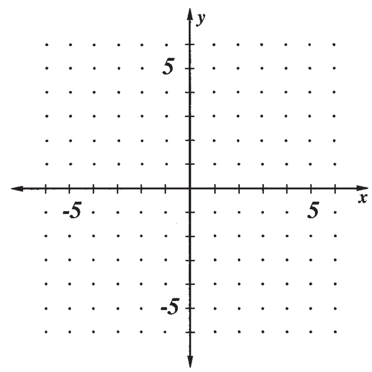
8. **The figure at the right is made from a parallelogram $ABCD$, and equilateral triangle ΔCDE. Find the $m\angle ADE$.**



1. a. Use the equation $x^2 + y^2 = 25$ and complete the table.

   Remember to include the plus and minus values.

$x =$	-5	-4	-3	0	3	4	5
$y =$							



   b. Graph all points on the coordinates at the right and connect with a smooth curve.

   c. Identify this geometric figure. ____________

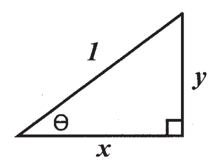
2. From eye level five feet above the ground, a person has to look up at an angle of 40° to the top of a flagpole 45 feet away. (Measure distance along the horizon.)

   a. Draw and accurately label a sketch of this.

   b. Find the height of the flagpole.

3. Use the right triangle pictured at the right.
   Find the following:

   a. $\sin\Theta =$ b. $\cos\Theta =$ c. $\tan\Theta =$



   Use this information to prove two identities.

   d. Since $y = \sin\Theta$ and $x = \cos\Theta$, by substitution $\tan\Theta = \underline{\qquad}$.

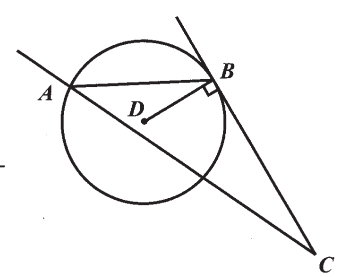
   e. By the Pythagorean theorem you know $x^2 + y^2 = 1^2$ or $x^2 + y^2 = 1$.

   Substitute for x and y to find the Pythagorean identity: ______________.

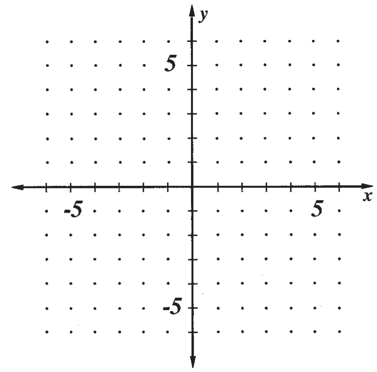
   Note: $\sin^2\Theta = (\sin\Theta)^2$.

4. Use the figure pictured at the right. Identify the following lines:

   a. tangent _____ b. chord _____ c. secant _____



1. a. Graph ΔABC where $A = (-1, 1)$ $B = (-1, -3)$ $C = (3, 1)$.



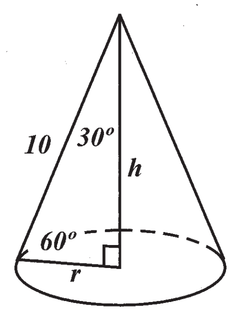
b. Find the coordinates of the midpoint of $\overline{AB}$ and label it D.

c. Find the coordinates of the midpoint of $\overline{BC}$ and label it E.

d. Verify that $\overline{AC} \parallel \overline{DE}$.

2. Given: the right cone pictured below with edge = 10 units.
To the nearest tenth, find the following:

a. The height (h) of the cone.



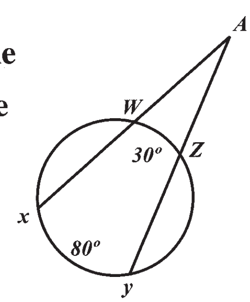
b. The radius (r) of the cone.

c. The surface area of the cone.

d. The volume of the cone.

3. Recall $\sin^2\Theta + \cos^2\Theta = 1$. If Θ is an acute angle and $\sin\Theta = .8$, find $\cos\Theta$.

4. The angle-secant theorem states that the measure of an angle formed by two secants intersecting outside a circle is half the difference of the arcs intersected by the angle.
$m\overparen{XY} = 80°$ and $m\overparen{WZ} = 30°$ find $m\angle A$



Point, line plane postulate:

a. Two points determine exactly one line.
b. Every line consists of points that can be put into a one-to-one relationship with the real numbers.
c. A point has no dimension.
A line has one dimension.
A plane has two dimensions.
d. Two different lines intersect in at most one point.

Triangle inequality portulate: The sum of the lengths of two sides of a triangle is greater then the length of the third side.

Postulates of logic:

a. Law of detachment: Given the true statement $p \Rightarrow q$, then given p one can conclude q.

b. Law of transitivity: Given the true statements $p \Rightarrow q$ and $q \Rightarrow r$, then given p one can conclude r.

c. Law of the contrapositive: Given the true statement $p \Rightarrow q$, one can conclude $\text{not - } q \Rightarrow \text{not - } p$.

d. Law of ruling out possibilities: If p or q is true and p is false, then q is true.

e. Law of indirect reasoning: If p or not-p is true, and p is proven false, then not-p is true.

Linear pair theorem: If two angles form a linear pair, then they are supplementary.

Vertical angles theorem: If two angles are vertical angles, then they are equal.

Parallel lines and slopes theorem: If two lines are parallel, then they have the same slope.

Perpendicular lines and slopes theorem: Two nonvertical lines are perpendicular if the product of their slopes is -1.

Transitivity of parallelism: If two lines are parallel to the same line, then they are parallel to each other.

Perpendicular to parallels theorem: If a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.

CPCF theorem: If two figures are congruent, then their corresponding parts are congruent.

A-B-C-D theorem: Every isometry preserves angle measure, betweenness, collinearity, and distance.

Reflexive property of congruence: Every figure is congruent to itself.

Transitive property of congruence: If $A \cong B$ and $A \cong C$ then $B \cong C$. Or, two figures congruent to the same figure are congruent to each other.

|| lines $\Rightarrow$*AIA* $\cong$: If two parallel lines are intersected by a transversal, then the alternate interior angles are equal.

acute angle: Angle measuring greater than 0° and less than 90°.

adjacent angle: Two nonstraight and nonzero angles with a common side interior to the angles formed by the uncommon sides.

algebraic equation: A math sentence relating two expressions as equal.

altitude: The perpendicular distance from the vertex of a triangle to the side opposite. Also, the perpendicular distance between parallel lines.

angle: The union of two rays, the sides, at a point, the vertex.

angle bisector: A ray that divides an angle into two equal parts.

angle of depression: The angle measured from the horizontal downward fro the observer's eye.

area: The number of unit squares or parts of unit squares required to tile a plane figure.

ex. parallelogram: $A = bh$

rectangle: $A = lw$

triangle: $A = \frac{1}{2}bh$

trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$

circle: $A = \pi r^2$

base: The variable b in the expression b^n.

bisector: A point, line, ray, or plane that divides a segment, angle, or figure into two parts of equal measure.

circle: The set of all points, the radius, equal distance from a certain point, the center.

circumference: The perimeter of a circle. $c = \pi d$. The ratio of the circumference to the diameter is π.

collinear points: Points that line on the same line.

complementary angles: Two angles whose sum is 90°.

cone: The surface of a conic solid whose base is a circle.

congruent figures: Two figures, *A* and *B*, are congruent if one is the image of the other under a reflection or a composite of reflections.

conic solid: The set of points between a given point, the vertex, and the points in a given region, the base, and the vertex and the base.

construction: A drawing make using a straight edge and compass and following a set of rules.

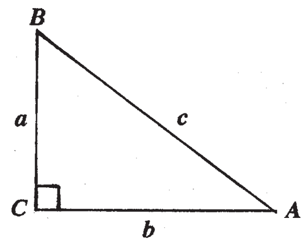
converse: A statement formed by switching the antecedent and the consequent of a given statement.

coordinates: An ordered pair.

PYTHAGOREAN THEOREM: $\{c^2 = a^2 + b^2\}$ **The Pythagorean theorem states that, in a Right Triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides (called legs). Whenever you have two of the three lengths, then you can find the third.**

example: Given $a = 4$, $b = 3$, find the hypotenuse.

$$c^2 = a^2 + b^2$$
$$c^2 = 4^2 + 3^2$$
$$c^2 = 16 + 9$$
$$c^2 = 25$$
$$c = 5$$



DISTANCE FORMULA:. Distance is the absolute distance between two points. In the coordinate system, it can be found using: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **.**

Notice that the distance formula is very closely related to the Pythagorean theorem.

$$c^2 = a^2 + b^2$$
$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2, \quad \text{so} \quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

example: Find the distance between (-3, 5) and (5, -6).

$$d = \sqrt{(5 - -3)^2 + (-6 - 5)^2}$$
$$d = \sqrt{8^2 + (-11)^2}$$
$$d = \sqrt{185}$$
$$d \approx 13.6$$

SLOPE: Slope is the rate of change. It is represented by the variable m.

On a graph $m = \dfrac{y_2 - y_1}{x_2 - y_1} = \dfrac{\text{change in } y}{\text{change in } x} = \dfrac{\text{rise}}{\text{run}}$

example: Find the slope of the line through (-3, 5) and (5, -6).

$$m = \frac{-6 - 5}{5 - -3}$$
$$m = \frac{-11}{8}$$

For every horizontal move of 1 unit to the right, there is a vertical change of $-\frac{11}{8}$**.**

Taking this one step further, when an equation is written in the slope/intercept form, $y = mx + b$**, the m, which is the coefficient of the x, is the slope.**

SOLUTIONS

pg. 1

① zero, one, two

② a) 

③ $-15+8=-7$ or $-15-8=-23$ · b) ray

④ a. $C=2\pi r = 2\cdot\pi\cdot 10 = 20\pi$ cm.
b. $A=\pi r^2 = \pi\cdot 10^2 = 100\pi$ cm²

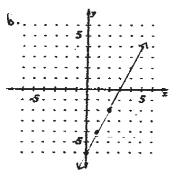
⑤ $15.6-8.4=7.2$

⑥ $5x+20=2x+56$
$3x=36$
$x=12$

$5(12)+20=80$
$m\angle 2 = m\angle 4 = 80°$
$m\angle 1 = m\angle 3 = 100°$

⑦ $(-2,5) \rightarrow (-2+2, 5-6) \rightarrow (0,-1)$

⑧ a. $2x-y=6$
$-y=-2x+6$
$y=2x-6$

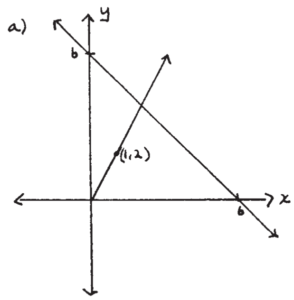
b. 

⑨ $x^2+5x+2x+10$
$x^2+7x+10$

⑩ a) $s \cap t = \{\overline{AB} \cup \overline{BC}\}$ b) $s \cup t = \{ABCD \cup \overline{AC}\}$

pg. 2

① a) $\{2,4\}$ b) $\{1,2,3,4,5,6,8,10\}$

② a) 

b) Intersection is a point (2,4)

③ $14+15<30$ No

④ $k=\frac{1}{4}$ $k^3=\frac{1}{64}$ the weight $k^2=\frac{1}{16}$ the area

⑤ Ingredients is the area, so if $k=\frac{16}{10}=\frac{8}{5}$, then $k^2=\frac{64}{25}$

$\frac{64}{25}(9.80) \approx \$25.00$

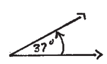
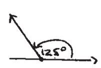
⑥ $-x+60+6x+5=180$
$5x+65=180$
$5x=115$
$x=23$
$-23+60=37°$
$6(23)+5=143°$

⑦ $A'=(2,3)$ $B'=(-2,1)$ $C'=(0,-3)$

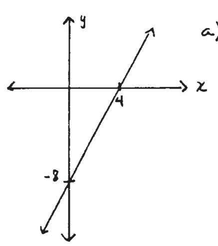
⑧ a) $\frac{3}{5}=\frac{x}{230}$ b) $5x=690$
$x=138$ girls

pg. 3

① $m\angle 3 = m\angle 1 = 57°$
$m\angle 4 = m\angle 2 = 123°$
Vertical angles are equal. If two angles form a linear pair then they are supplementary.

② a.  b. 

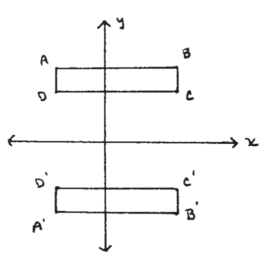
③ $0<23+x<90$
$-23<x<67$

④ 

a) (4,0) b) (0,-8)

⑤ Let x = smaller angle
$3x$ = larger angle
$x+3x=90$
$4x=90$
$x=22.5°$ $3x=67.5°$

⑥ If two lines are in the same plane then they intersect.
False. The may be parallel.

⑦ 

⑧ 12 ft = 144 in. 16 ft = 192 in
$A = 144 \times 192 = 27648$ in²
$A_{brick} = 32$ in² $\frac{27648}{32}=$
864 bricks

⑨ $3^2 = 9$ times

a) $A'=(-2,-3)$ $B'=(3,-3)$ $C=(3,-2)$ $D=(-2,-2)$
c) rectangle
d) $A=lw=5\cdot 1=5$ sq. units

pg. 4

① parallel ② parallel

③ $.8+.9=1.7$ $1.7>1$
$.8+1=1.8$ $1.8>.9$
$.9+1=1.9$ $1.9>.8$

Yes. The sum of any two sides is greater than the 3rd. side.

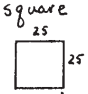
④ ∠B. It is opposite the longest side.

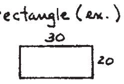
⑤ $\frac{.5}{1000}=\frac{1}{2000}=\frac{\text{model}}{\text{bridge}}$ or $\frac{2000}{1}=\frac{\text{bridge}}{\text{model}}$

⑥ a. $6y-13=4y+3$
$2y-13=3$
$2y=16$
$y=8$

b. $m\angle ABD = m\angle CBD = 35°$
$6(8)-13=35°$

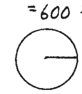
⑦ square $\frac{100}{4}=25$ rectangle (ex.)





$A=25^2=625$ ft² $A=20\cdot 30 = 600$ ft²

circle $100=\pi d$
$d \approx 31.8$
$r \approx 15.9$



$A=\pi r^2 = \pi\cdot 15.9^2 \approx 794$ ft²

The circle has the largest area, but it might not be the most practical shape.

⑧ $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$d=\sqrt{3^2+1^2}=\sqrt{10}$

⑨

Conclusions	Justifications
① $m\angle 1 = m\angle 2$	① Given
② $m\angle 1 = m\angle 3$	② Vertical angles are =.
③ $m\angle 2 = m\angle 3$	③ Transitive property of equality