Algebra Curriculum Overview

The Number System

- Find common factors and multiples.
- Evaluate absolute values.
- Write numbers in scientific notation.
- Perform operations of multiplication and division on numbers written in scientific notation.
- Change a rational number to a decimal and/or a percent.
- Understand that an irrational number can be approximated by a rational number.
- Evaluate a factorial.
- Use the order of operations to simplify an expression.
- Evaluate expressions containing radicals.
- Evaluate expressions containing variables to powers.
- Understand and use the Pythagorean Theorem.
- Understand the triangle inequality.
- Use the distance formula to find the distance between two points.
- Find the slope of a line using two points.
- Graph a number sentence on an number line.
- Identify the subsets of real numbers as rational or irrational.
- Know that some numbers are not real (note: Imaginary numbers are identified in a higher course).

Seeing Structure in Expressions

- Write an expression in algebraic language.
- Identify the parts of an expression (e.g. terms, factors, coefficients).
- Identify ways to rewrite expressions {e.g. $a^2 - b^2 = (a - b)(a + b)$ }.
- Factor quadratic expressions.
- Complete the square of quadratic expressions.
- Use the properties of exponents.
- Find a rule for an arithmetic series.

Arithmetic with Polynomials and Rational Expressions

- Perform arithmetic operations on polynomial expressions.
- Identify the zeros of a polynomial when factorization is possible.
- Simplify rational expressions.
- Understand that the operations used on polynomials are analogous to those used on integers.

Creating Equations

- Create equations and inequalities in one variable.
- Create equations in two variables.
- Graph equations on the coordinate system.
- Rearrange a formula to highlight a quantity of interest (e.g. $d = rt$ can be renamed as $r = \frac{d}{t}$).
- Find an equation for a line given two points or a point and the slope.
- Identify the domain and range of a function.
- Evaluate equations in function notation.

Reasoning with Equations and Inequalities

- Solve simple rational and radical equations.
- Solve equations and inequalities in one variable.
- Justify each step in solving a linear equation.

Reasoning with Equations and Inequalities (continued)

- Solve a system of equations using graphing.
- Solve a system of equations using substitution.
- Solve quadratic equations by taking the square root, by factoring, by completing the square, by using the quadratic formula and by factoring.
- Use the discriminant test to determine if the roots of a quadratic equation are real.
- Graph inequalities on a number line and on the coordinate system.
- Solve a system of inequalities by graphing.
- Understand and apply the Pythagorean Theorem.

Statistics and Probability

- Find theoretical probabilities.
- Find the mean and median of a data set.
- Answer questions from a dot frequency graph.

Graphics Calculator

- Understand the basic operations.
- Use the function keys.
- Use the edit keys.
- Create graphs.
- Solve complex problems.
- Solve systems of equations.
- Find the maximum of a function.
- Find the zeros of a function.

Algebra 1

Instructions for the User

- To use Summer Math Skills Sharpener, simply tear off a page and complete it. The program is designed to be used 3 days per week for 10 weeks.
- Supplemental lessons have been included to address changing state and National Standards. These lessons are OPTIONAL. (See green supplement following body of program.)
- Detailed solutions to all the problems are included at the back of the book. Please complete an entire sheet prior to checking your answers. You may want to remove the solutions and store them in a place apart from the book.
- All concepts are part of a standard algebra 1 curriculum. Please attempt all problems. In addition to the solutions, pink "Help Pages" have been included to assist you in completing the problems.
- A yellow "Glossary of Terms" is located at the back of the book.
- Pages should be worked in order. While each page contains mixed concepts, individual concepts, within the book, have been ordered from easier to more difficult.
- If you experience difficulty with certain concepts, address the problem with your teacher. He or she may recommend additional help in these areas.
- It is important to give every problem your best effort. Problems may seem challenging, but use a combination of the "Help Pages" and the "Solutions" to assist you for maximum success.
- We appreciate your comments. Please complete the enclosed evaluation page after you have entered your next math course but before November 1st.

For problems 1 – 3, solve for x.

1. $x - 7 - 4x = -4$
2. $2(x - 1) - 7 = 5$
3. $7 - 4(x + 1) = -1$

4. **The perimeter of a rectangular lot is 156 meters. The length is twice the width.**

   a. **Write the equation that describes this situation.**

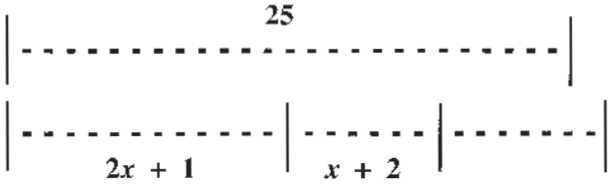
   b. **Find the length and the width.**

5. **The perimeter of an equilateral triangle is 7.5 inches. What is the length of one side?**

6. **Evaluate $3(k - j)$ where $k = \frac{1}{3}$ and $j = \frac{4}{9}$.**

7. **(8, 3) and (5, y) lie on the same line. If the slope of the line is $\frac{1}{2}$, find y.**

8. **Write the inequality to represent the diagram.**



9. **Graph $x \geq -2$.**

10. Multiply each of the following:

a. $(x-1)(2x^2+x+5)$

b. $(x+3)(x-3)(2x+1)$

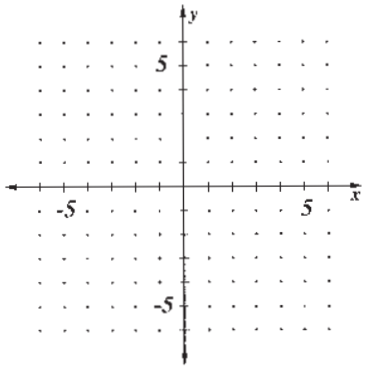
11. Find a number between $\frac{9}{16}$ and $\frac{21}{32}$.

12. A euchre deck of cards consists of 24 cards. It includes the 9 through ace of each suit. In a particular game, what is the probability that the first two cards dealt will be first the jack of spades and then the jack of clubs?

13. a. Find three pairs of numbers that satisfy the equation $y = 2x - 3$.

b. Plot these points on the graph at the right and connect them with a line.

c. When an equation is in the form $y = mx + b$, m represents the slope of the line. What is the slope of this line?



d. Choose two of your points and verify this.

For problems 1 – 4, solve for x.

1. $\sqrt{x} = 25$ 2. $x^2 = 25$ 3. $|x| = 25$ 4. $|x+1| = 25$

5. **In 2000 Lincoln High School had 804 students. It has been gaining about 30 new students each year. In that same year Washington High School had 1,008 students. It has been gaining only about 10 new students each year.**

   a. **Write an equation which states that after x years both schools will have the same number of students.**

   b. **Solve for x. In what year will the schools have the same number of students?**

6. **Factor each of the following:**

   a. $x^2 + x - 6$ b. $x^2 - 49$ c. $x^2 + 26x + 25$

7. **There are twelve players on the varsity basketball team. How many different ways can they line up for a team picture?**

8. **A slope of −3 means that for every horizontal change of 1, there is a vertical change of _____.**

9. The circumference of a circle (C) is given by the formula $C = 2\pi r$ or $C = \pi d$ where r is the radius and d is the diameter.

   a. Find the exact circumference of a circle with $d = 8$ units.

   b. Approximate the circumference to the nearest tenth.

   c. Find the exact area of the same circle.

   d. Approximate the area to the nearest tenth.

10. The equation $ax^2 + bx + c = 0$ can be solved for x using the *quadratic formula* or $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The *discriminant* ($D$) is $b^2 - 4ac$. Always calculate this first. Then you can solve $x = \dfrac{-b \pm \sqrt{D}}{2a}$.

    Solve $x^2 + 7x + 12 = 0$

    a. Find the discriminant.

    b. Find x.

11. a. Find the slope of the line determined by $(-3, 3)$ and $(-4, -1)$.

    b. Using the language of problem 8, state what this means.

For problems 1-3, simplify.

1. $(x-5)(2x+8)$ 2. $(x^2-2x+3)(x+2)$ 3. $(x^2-3x+9)(x+3)$

For problems 4-5, simplify by first writing as a complex fraction. Recall that "*per*" means division. Label with the correct units.

Example:

$$\frac{\$3.36 \text{ per dozen}}{12 \text{ cookies per dozen}} = \frac{\dfrac{\$3.36}{1 \text{ dozen}}}{\dfrac{12 \text{ cookies}}{1 \text{ dozen}}} = \frac{\$3.36}{1 \text{ dozen}} \bullet \frac{1 \text{ dozen}}{12 \text{ cookies}} = \$.28 \text{ per cookie}$$

4. $\dfrac{8000 \text{ miles per hour}}{200 \text{ gallons per hour}}$ 5. $\dfrac{150 \text{ heartbeats per min}}{0.25 \text{ miles per min}}$

6. **If the area of a rectangle is 24 square units, its perimeter is 20 units.**
   a. **Draw and label an accurate instance of this conditional.**

   b. **Draw and label a counter example to this conditional.**

7. **Provide the justifications for each step in the simplification below.**

$2(a+3)+4(a-3)$	Given
$2a+6+4a-12$	_____
$2a+4a+6-12$	_____
$6a-6$	_____

8. **The sum of the angles of a triangle is 180°. Figure *ABC* is a triangle. You conclude that the sum of the angles is 180°.**
   a. **Is this an example of deductive or inductive reasoning?**
   b. **Justify your answer.**

9. a. You mix 20 chocolate covered raisins with 35 peanuts. What percent of your mix is chocolate covered raisins?

   b. A box contains 26 pens and pencils. If $x\%$ of the mix is pencils, how many pencils are in the box?

   c. You mix x orange jelly beans with 25 black jelly beans. What percent of your mix is orange jelly beans?

10. Each fall Justin and Tyrone rake leaves to earn extra money. Justin can rake an average yard in 5 hours. Tyrone can rake an average yard in 6.5 hours. How long will it take them to rake a yard if they work together?

   a. What part of a yard can Justin rake in one hour?

   b. What part of a yard can Tyrone rake in one hour?

   c. What part of a yard can they rake in one hour if they work together? (This is the sum of your answers to a and b.)

   d. Let t = the time it takes to do the job together. In terms of t what part of the yard can they complete in one hour?

   e. Using your answers to "c" and "d", find how long it will take them to rake the yard if they work together.

11. Find an equation for the line parallel to $y = -2x + 4$ containing $(2, 5)$.

acute angle : an angle measuring greater than 0 degrees and less than 90 degrees.

algebraic equation : a math sentence relating two expressions as equal.

algebraic expression : a combination of numbers and variables.

altitude : the perpendicular distance from the vertex of a triangle to the side opposite. Also, the perpendicular distance between parallel lines.

angle : the union of two rays (the sides) at a point (the vertex).

area : the number of unit squares or parts of unit squares required to tile a plane figure.

ex. parallelogram : $A = hb$

rectangle : $A = lb$

triangle : $A = \frac{1}{2}hb$

trapezoid : $A = \frac{1}{2}h(b_1 + b_2)$

circle : $A = \pi r^2$

arithmetic sequence : sequence with a constant difference.

base : the variable b in the expression b^n.

binomial : a polynomial containing two terms.

circle : the set of all points (the radius), equal distance from a certain point (the center).

circumference : the perimeter of a circle. $c = \pi d$. The ratio of the circumference to the diameter is π.

complementary angles : two angles whose sum is 90°.

composite of functions : an operation that first applies one function, then the other. It is written $f(g(x))$ or $f \circ g(x)$. Both mean f following g of x.

coordinates : an ordered pair.

ORDER OF OPERATIONS: Parentheses, powers, multiplication, division, addition, subtraction.

{use "Pretty Please, My Dear, Aunt Sally" to help remember}

ex. $(x-2x)^2+2x^2-3(x+3x)$

1. Parentheses first : $(-x)^2+2x^2-3(4x)$

2. Powers second : $x^2+2x^2-3(4x)$

3. Multiplication and/or Division third : x^2+2x^2-12x

4. Addition and/or Subtraction of LIKE terms last : $3x^2-12x$

PYTHAGOREAN THEOREM : $\{c^2=a^2+b^2\}$ **The Pythagorean theorem states that, in a RIGHT triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides (called legs). Understanding this is essential to a complete understanding of right triangles, similar triangles, and trigonometry. Whenever you have two of the three lengths, then you can find the third.**

ex. Given hypotenuse $c=13$, **leg** $b=5$, **find leg** a.

$c^2=a^2+b^2$

$13^2=a^2+5^2$

$169=a^2+25$

$144=a^2$

$a=12$

SLOPE : Slope is the rate of change. It is represented by the variable m.

On a graph $m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{change\ in\ y}{change\ in\ x}=\dfrac{rise}{run}$

A popular description is to think of slope as the amount of vertical change for every horizontal move of 1 unit to the right.

ex. Find the slope of the line through (1, - 1) and (3, 5)

$m=\dfrac{y_2-y_1}{x_2-x_1}$

$m=\dfrac{5-(-1)}{3-1}$

$m=\dfrac{6}{2}=3$

For every horizontal move of 1 unit to the right, there is a vertical change of + 3.

Taking this one step further, when an equation is in the slope/intercept form, $y=mx+b$, **the** m, **which is the coefficient of the** x, **is the slope.**

Lesson 7 pg. 13

① $|x|-2=3$
$+2\ +2$
$|x|=5$
$x=5$ or $x=-5$

② $|x-2|=3$
$x-2=3$ or $x-2=-3$
$+2\ +2$ $\quad +2\ +2$
$x=5$ or $x=-1$

③ $x^2-2=7$
$+2\ +2$
$x^2=9$
$x=\pm 3$

④ $(x-2)^2=9$
$x-2=3$ or $x-2=-3$
$+2\ +2$ $\quad +2\ +2$
$x=5$ or $x=-1$

⑤ 4000 miles $\cdot \frac{5280\text{ ft.}}{\text{mile}} = 21120000$ ft.

30 yrs. $\cdot \frac{365\text{ days}}{\text{yr}} = 10950$ days

$\frac{21120000\text{ ft.}}{40\text{ ft.}} = 528000$ lengths

$\frac{528000\text{ lengths}}{10950\text{ days}} \approx 48.2$

Assuming she never missed a day she swam just over 48 laps each day.

⑥ $A=lw$ $\quad (x+6)(x+3)=x^2+9x+18$ units^2

⑦ sum is 10 product is 16

⑧ a. $4\cdot3\cdot2=24$ b. $\frac{1}{24}$ There are 24 possible outcomes. Only one satisfies the event.

⑨ a. $4!=4\cdot3\cdot2\cdot1=24$ b. $8!-2!=$
$8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1-2\cdot1=40318$
c. $4(3!)=4\cdot3\cdot2\cdot1=24$ d. $0!=1$ by definition

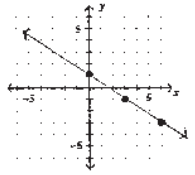
Lesson 7 continued pg. 14

⑩ a. $A=\pi r^2=\pi\cdot3^2=9\pi$ units^2 b. $\frac{A}{\pi}=\frac{\pi r^2}{\pi}$
$r^2=\frac{A}{\pi}$
$r=\sqrt{\frac{A}{\pi}}$
c. $r=\sqrt{\frac{78\pi}{\pi}}$ $\quad r=\sqrt{78}\approx 8.8$ units

⑪ a. $P=2(l+w)$
$=2(y+8+y)$
$=2(2y+8)=4y+16$ units
b. $A=lw$
$=y(y+8)$
$=y^2+8y$ units^2

⑫ a. It's easiest to do a first and solve for y in terms of x.
$2x+3y=3$
$-2x \quad -2x$
$\frac{3y}{3}=\frac{-2x+3}{3}$
$y=-\frac{2}{3}x+1$

In order to have integer values, choose 0, 3, and multiples of 3.
$y=-\frac{2}{3}(0)+1=1$ $\quad (0,1)$
$y=-\frac{2}{3}(3)+1=-1$ $\quad (3,-1)$
$y=-\frac{2}{3}(6)+1=-3$ $\quad (6,-3)$



c. See a. $y=-\frac{2}{3}x+1$

d. $m=-\frac{2}{3}$ $\quad b=1$

Lesson 8 pg. 15

① $\sqrt{x}=25$
$(\sqrt{x})^2=25^2$
$x=625$

② $x^2=25$
$x=\pm 5$

③ $|x|=25$
$x=25$ or $x=-25$

④ $|x+1|=25$
$x+1=25$ or $x+1=-25$
$-1\ -1$ $\quad -1\ -1$
$x=24$ or $x=-26$

⑤ a. $804+30x=1008+10x$

$2000+10.2=2010.2$
In the year 2011

b. $804+30x=1008+10x$
$-10x \quad -10x$
$804+20x=1008$
$-804 \quad -804$
$\frac{20x}{20}=\frac{204}{20}$
$x=10.2$ yrs.

⑥ a. x^2+x-6
$(x+3)(x-2)$

b. x^2-49
$(x-7)(x+7)$

c. $x^2+26x+25$
$(x+25)(x+1)$

⑦ $12!=12\cdot11\cdot10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1=479{,}001{,}600$ ways

⑧ Vertical change of -3.

Lesson 8 continued pg. 16

⑨ a. $C=\pi d$
$=\pi\cdot8=8\pi$ units

b. 25.1 units

c. $A=\pi r^2$
$=\pi\cdot4^2=16\pi$ units^2

d. 50.3 units^2

⑩ a. $a=1$ $\quad b=7$ $\quad c=12$
b^2-4ac
$7^2-4(1)(12)=1$

b. $x=\frac{-7\pm\sqrt{1}}{2}$
$x=\frac{-7+1}{2}=-3$
$x=\frac{-7-1}{2}=-4$

⑪ a. $m=\frac{y_2-y_1}{x_2-x_1}$

$m=\frac{-1-3}{-4-(-3)}=\frac{-4}{-1}=4$

b. For every horizontal change of 1 unit there is a vertical change of 4.