

Basic Math Review for the Middle Grades Curriculum Overview

Number Sense

- Add, subtract, multiply and divide with fractions and decimals.
- Order a set of fractions, percents and decimals from least to greatest.
- Compare numbers using the symbols for greater than, less than or equal to.
- Identify prime numbers.
- Write a number using exponents.
- Compare place value (e.g., how many times larger is the 5 in .051 than the 5 in .625).
- Add and subtract with positive and negative numbers.
- Approximate positive and negative numbers on a number line.
- Round numbers in a range from ten million to ten thousandths.
- Estimate using addition, subtraction, multiplication and division.
- Determine equivalent decimals (e.g., $0.8 = 0.80$).
- Find a fraction that is closer in value to a given percent.
- Find the equivalent values of a fraction, decimal, percent (e.g., $7\% = .07 = 7/100$).
- Find the number in a set of fractions, decimals and percents that is not equivalent to the others.
- Use mental math to multiply and divide fractions (e.g., $8\frac{1}{2} \times 7/7$).
- Use mental math to halve and double numbers.
- Find the square root of a given number.
- Determine the greatest common divisor, least common denominator, least common multiple of a given set of numbers.

Measurement

- Convert within a system of measurement (e.g., 86 ounces = 5 pounds 6 ounces).
- Determine the amount of elapsed time from one time to another.
- Determine the time after adding hours and minutes.
- Determine time using more than one time zone.
- Determine the fraction and the percent of an hour given the number of minutes.
- Determine the balance in a checking account.
- Determine the price per unit given the total cost and the total number of units.
- Estimate the price of a sale item given the percent discount.
- Determine the percent discount given the original price and the sale price.
- Determine the total price of several items after calculating the sales tax.
- Determine the best buy.
- Determine the total price of a restaurant bill after calculating the tip.
- Determine the percent of a dollar given a set of coins (e.g., 5 quarters = 125% of a dollar).
- Determine averages.

Expressions and Equations

- Identify the radius, diameter and center of a circle.
- Find the circumference of a circle given the diameter.
- Determine the radius of a circle given the area.
- Determine the area of a quadrilateral and a triangle.
- Determine the third angle of a triangle given the other two angles..
- Determine the base of a triangle given the height and the area.
- Determine the perimeter of a quadrilateral given the length of one side and the area.
- Determine the length of the sides of a square given the area.
- Determine the area of a square after increasing the sides.
- Draw a geometric figure given the information of the angles and the sides (e.g., four congruent sides, two acute angles and two obtuse angles).
- Identify angles.
- Find the complement and supplement of given angles.

Expressions and Equations (continued)

- Find the volume of a cube.
- Locate points on a grid given the ordered pairs

Data Analysis, Statistics and Probability

- Determine the probability of an event (e.g., your telephone number ending in a 5, today being your birthday in a non leap-year).
- Interpret information from charts and graphs (line, bar, and circle).

Prealgebra Lessons

- Recognize and use the math verbs correctly in a math sentence.
- Explore the powers of 10, both positive and negative, by moving the decimal place accordingly.
- Demonstrate an understanding of terminating and non-terminating decimals.
- Evaluate expressions using the rules for the order of operations.
- Use the property of reciprocals to divide fractions.
- Describe rate as a fraction of two numbers with different units.
- Describe ratio as a fraction of two numbers with the same units.
- Demonstrate an understanding of the distributive, commutative and associative properties.
- Use scientific notation to represent large and small numbers.
- Describe a pattern using variables.
- Name and draw points on the xy-coordinates.
- Graph the solution to an equation on the xy-coordinates.
- Show slope as a rate of change.
- Calculate the slope given two points on a line or the equation for a line.
- Calculate the distance between two points.
- Graph linear, quadratic and exponential functions.
- Draw the image of a polygon under a translation or a reflection.
- Identify the legs and the hypotenuse of a right triangle.
- Translate words to algebraic expressions or equations.
- Create and apply an algebraic equation to solve a real life situation.
- Justify responses to problems.
- Find the next term, the constant difference and a rule for an arithmetic sequence.
- Make conclusions from a stem-and-leaf plot.
- Identify the range, mean, median and mode of a data set and explain the effect of an outlier.
- Create a box-and-whisker plot from a data set.
- Recognize that a sample may be biased.
- Solve probability problems that contain “or” and “and.”
- Solve algebraic equations.
- Find the third side of a right triangle using the Pythagorean Theorem.
- Demonstrate an understanding of absolute value.
- Demonstrate an understanding of the area formulas.
- Simplify a polynomial.
- Determine the number of terms, the degree, and the constant for a polynomial.
- Solve simple interest problems.

IMPORTANT!
Instructions for Parents

- To use Summer Math Skills Sharpener, simply tear off a page and have your child complete it. The book is designed to be used 3-4 days per week for 10 weeks.
- Check answers immediately for optimal feedback. An answer sheet and solution pages are provided at the back of the book. (**Solution pages represent only ONE method of solving each problem.**)
- “Help Pages” have been added at the front of the book to clarify certain concepts.
- Allow your child to use a calculator only for those problems marked “Calculator Skill.”
- When solving division problems with remainders, students should use fractions or decimals rounded off to the nearest hundredth.
- When solving fraction problems, students should reduce answers to the lowest possible terms.
- Pages entitled “Brain Aerobics” are located at the back of the book. Have your child complete one page per week for extra practice. The answers to these questions are on the back of each page.
- A “Glossary of Terms” and a “Table of Measurements” are at the back of the book.
- Adjust the program to vacations, etc. Presentation of mixed concepts on every page ensures that all skills are reinforced; therefore, pages may be completed in any order.
- If your child experiences difficulty with concepts that have been already taught, address the problem with his or her teacher in the fall; more consistent problems indicate that a tutor may be needed.

We appreciate your comments. Please send in the enclosed evaluation page before November 1st, after your child has returned to school in the fall and you are able to determine the success of the Summer Math Skills Sharpener.

Dividing a Whole Number by a Fraction

Example: $2 \div \frac{1}{3}$

1st Step $2 = \frac{2}{1}$

Put the *whole number* in fraction form.

2nd Step $\frac{2}{1} \div \frac{1}{3}$ ← divisor

Locate the divisor and invert.

$\frac{1}{3}$ becomes $\frac{3}{1}$

3rd Step $\frac{2}{1} \times \frac{3}{1} = \frac{6}{1}$

Invert the divisor and multiply.

4th Step $\frac{6}{1} = 6$

Dividing a Fraction by a Whole Number

Example: $\frac{1}{3} \div 2$

1st Step $2 = \frac{2}{1}$

Put the whole number in fraction form.

2nd Step $\frac{1}{3} \div \frac{2}{1}$ ← divisor

Locate the divisor and invert.

$\frac{2}{1}$ becomes $\frac{1}{2}$

3rd Step $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

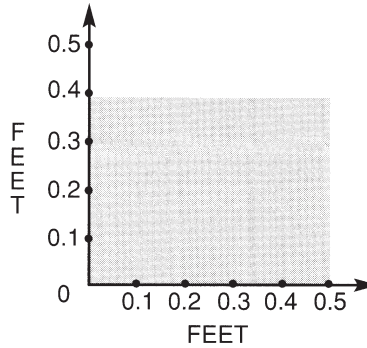
Multiply the numerators and denominators.

4th Step $\frac{1}{6}$ is in lowest terms

Simplify (reduce) if necessary.

1. a. Change 60% to a fraction. _____
- b. Change $\frac{9}{20}$ to a percent. _____

2. Find the area of this shaded rectangle:



3. Find the average of these numbers: 17, 13, 10, 16, 14

4. Hershel had 100 baseball cards that he labeled from 1-100. He started with number one and marked every 5th card with an X, every 7th card with an O and every 10th card with a \checkmark . What number card will be the first to have all 3 marks (XO \checkmark)?

5. How many games are played in a 4 team round robin tournament? (Each team plays every other team only once.)

6. Find the value of n: $2,000 - n = -2,000$ $n =$ _____

7. Which is closest to 35% of 280? _____

a. $\frac{1}{2}$ of 280

b. $\frac{1}{3}$ of 280

c. $\frac{1}{4}$ of 280

8. a. $3\frac{1}{4} \div \frac{1}{4} =$ _____

b. $7\frac{1}{5} \times \frac{1}{2} =$ _____

9. a. $\frac{1}{1,000} \times \frac{1}{1,000} =$ _____

b. $\frac{1}{100} \times \frac{1}{100} =$ _____

10. Find the circumference of a circular field with a diameter of 16 yards.
(Let $\pi = 3.14$)

11. Lana has a friend who is deaf and uses sign language to communicate. She can sign about 36 concepts per minute. How many concepts can she sign in a 15 minute period?

12. Calculator skill: Using a calculator, find the product of .42 and $\frac{7}{20}$.

7. What percent of a yard is:

a. 12 inches? _____

b. 18 inches? _____

c. 9 inches? _____

d. 3 inches? _____

8. $x + \frac{4}{5} = 1\frac{1}{4}$ $x =$ _____

9. Problem solving: Janine is 7 years younger than Lucy and 4 years older than Samantha. The average of their ages is 16. How old is:

a. Janine? _____

b. Samantha? _____

c. Lucy? _____

10. a. $\frac{3}{2} + \frac{5}{3} =$ _____

b. $(-14) - (-3) =$ _____

11. Calculator skill: $240^2 + 10.9^2 - .9^2 =$ _____

Turn your answer upside down to find out what you pay at the end of the month.

12. Susan drove 4 hours. Her average speed was 60 mph. Finish the chart below to give her an average speed of 60 mph.

Hour	Speed
1st	65
2nd	70
3rd	
4th	55

1. For problems, a – f, simplify. (Recall that the order of operations is: parentheses, powers, multiplication and division from left to right; addition and subtraction from left to right.)

a. $8 / 2 \times 3$

b. $4(6 - 2)^2$

c. $4 + 2(3 + 2)^2$

d. $\frac{1 - 1^5}{1^3}$

e. $9\left(\frac{2}{3}\right)^2$

f. $4 - 4 \times 2 - 2$

2. A *variable* is a symbol that can be replaced by a number.

Let $a = 3$ $b = -3$ $c = \frac{1}{3}$ $d = -\frac{1}{3}$

a. $a + b$

b. $a - b$

c. ab

d. $b \div d$

e. b^2

f. c^{-1}

3. An *arithmetic sequence* is a number pattern where the difference between consecutive numbers (terms) is constant.

The first three terms of a sequence are $-5, 1, 7, \dots$

a. Find the next two terms in this sequence. _____, _____

b. Find the difference (next term – previous term) between any two consecutive terms. _____

(This is the constant difference or the constant rate of change.)

c. If n is the number of the term in the sequence, find a rule or formula for any term in the sequence. (Hint: Find the zero term.)

d. Use your rule to find the tenth term in the sequence. _____

4. For problems a – c, solve for x .

a. $2x - 3 = 7$

b. $\frac{2}{3}x = 8$

c. $2x - \frac{3}{4} = \frac{7}{8}$

5. Use the *distributive property* to find the missing number or variable.

a. $7(x + y) = 7() + 7()$

b. $4x + 8y = ()(x + 2y)$

BASIC MATH REVIEW

BRAIN AEROBICS - WEEK 1

1. Maddy's mom decorated the clubhouse at their apartment complex for Maddy's birthday. She hung 2 strips of pink crepe paper and 3 strips of purple crepe paper all the way around the clubhouse room. The room measures 45 feet long and 30 feet wide. How many yards of crepe paper did Maddy's mom have to buy? If crepe paper costs \$3.00 for 25 yards, how much did she spend on crepe paper?
2. The veterinarian told Denise that her dog needed to lose some weight. She put him on a diet. Max lost 4% of his total weight in the first week. If Max weighed 48 pounds at the end of the first week, how much did he weigh before his diet?
3. Burns Park School lets basketball teams rent the gym on weekends for a fee of \$25.00 for $1\frac{1}{2}$ hour sessions. Last year the school earned \$11,400. For how many sessions was the gym rented?
4. One half of the students at Pioneer High School bring their lunches to school. One fourth of the students go home for lunch, and the remaining 500 buy their lunches. How many students attend Pioneer High?
5. It took Janine 20 minutes to knit $\frac{1}{8}$ of her scarf. How long will it take her to complete the entire scarf?

The Metric Units

Temperature - Celsius

0°C: the freezing point of water
37°C: the normal body temperature
100°C: the boiling point of water

Mass

1000 milligrams (mg) = 1 gram
1000 grams = 1 kilogram (kg)
1000 kilograms = 1 metric ton (t)

Capacity

1000 milliliters (mL) = 1 liter (L)
1000 liters = 1 kiloliter (kL)

Length

10 millimeters (mm) = 1 centimeter
10 centimeters (cm) = 1 decimeter
1000 millimeters (mm) = 1 meter (m)
100 centimeters = 1 meter (m)
10 decimeters (dm) = 1 meter
1000 meters (m) = 1 kilometer (km)

The Customary Units

Temperature - Fahrenheit

32°F: the freezing point of water
98.6°F: the normal body temperature
212°F: the boiling point of water

Weight

1 pound (lb) = 16 ounces (oz)
1 ton = 2,000 pounds

Time

1 minute (min) = 60 seconds (s)
1 hour = 60 minutes
1 day = 24 hours
1 week = 7 days
1 month (mo) = approx. 4 weeks
1 year (yr) = 365 days
52 weeks
12 months

1 decade = 10 years
1 century = 100 years

Capacity

1 cup (c) = 8 fluid ounces (fl oz)
1 pint (pt) = 16 fluid ounces
2 cups
1 quart (qt) = 32 fluid ounces
4 cups
2 pints
1 gallon (gal) = 128 fluid ounces
16 cups
8 pints
4 quarts

Length

1 foot (ft) = 12 inches (in)
1 yard (yd) = 36 inches
3 feet
1 mile (mi) = 5,280 feet
1,760 yards

GLOSSARY OF TERMS AND FORMULAS

a.m.: a way of expressing time between 12:00 midnight and 12:00 noon.

acute angle: an angle measuring less than 90 degrees.

addends: numbers to be added together in an addition problem.

algebraic expression: an expression that uses at least one letter to represent a number.

angle: the measured space between two lines that meet at one point.

area: the measure of a region in square units.

Formulas:

Quadrilateral: $A = \text{length} \times \text{width}$

Triangle: $A = \frac{1}{2} \times \text{base} \times \text{height}$

Circle: $A = \pi r^2$

$\pi \approx 3.14$

$r = \text{radius}$

average: a number obtained by adding a group of numbers together and dividing by the number of addends.

center: the point from which all points on a circle are equally distant.

circle: a closed, curved line. Each point on the circle is the same distance from the center of the circle.

circumference: the distance around a circle.

Formula: $c = 2 \pi r$ $r = \text{radius}$ $\pi \approx 3.14$

common denominator: a multiple of all denominators in a problem.

common factor: a number that is a factor of two other numbers is a **common** factor.

Example: 3 is a common factor of 9 and 12.

common multiple: a number that is a multiple of two other numbers.

Example: 24 is a common multiple of 6 and 4.

complementary angle: the number in degrees given to an angle that when added to another angle will give 90 degrees.

congruent: being the same.

coordinates: number pairs used in graphing. The horizontal axis is listed first and the vertical axis is listed second.

Example: 8, 10

customary measurement: units of measurement used in the United States.

Example: feet, pounds, miles, etc.

decade: a period of 10 years.

decimal point: a period put just before the decimal fraction such as the periods in .625 and .08

degree: a unit used to measure an angle or temperature.

1.) $(4^4) 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4,096$

2.)
$$\begin{array}{r} 845.000 \\ - .009 \\ \hline 844.991 \end{array}$$

3.) 3 angles of a triangle = 180°

$$\begin{array}{r} 140^\circ \\ + 37^\circ \\ \hline 177^\circ \end{array}$$

$$\begin{array}{r} 180^\circ \\ - 141^\circ \\ \hline 39^\circ \end{array}$$

4.) $17 \times 15 \times 10 = 2,550 \text{ cu. ft.}$
 Under by 450 cu. ft.

5.) a. $4\frac{1}{2} \div 3\frac{1}{2} = \frac{9}{2} \div \frac{7}{2} = \frac{9}{7}$
 $\frac{9}{7} \times \frac{7}{7} = \frac{9}{7} = (1\frac{2}{7})$

b. $\frac{100}{12} \times \frac{36}{36} = \frac{100}{1} = (14\frac{2}{7})$

7.) a. $\frac{12}{36} = \frac{1}{3} = (33\frac{1}{3}\%)$

b. $\frac{18}{36} = \frac{1}{2} = (50\%)$

c. $\frac{9}{36} = \frac{1}{4} = (25\%)$

d. $\frac{36}{36} = 1 = (8\frac{1}{3}\%)$

6.) $3 = b \div 3 \quad 3 \times 3 = 9$
 $1 = b \div 3 \quad 3 \times 1 = 3$

$-6 = b \div 3 \quad -6 \times 3 = (-18)$

$-12 = b \div 3 \quad -12 \times 3 = (-36)$

8.) $x + \frac{4}{7} = 1\frac{1}{7} = \frac{8}{7} = \frac{25}{20}$
 $-\frac{4}{7} \quad -\frac{4}{7} = -\frac{16}{20}$
 $x = \frac{9}{20}$

9.) Janine = J Lucy = J+7
 Samantha = J-4

$J + (J+7) + (J-4) = 48 \text{ yrs.}$

$(3J) + 7 - 4 = 48$

$3J + 7(-7) - 4 = 48(-7)$

$3J - 4 = 41$

$3J - 4(+4) = 41(+4)$

$3J = 45$

$3J \div 3 = 45 \div 3 = (15)$

$J = 15$

10.) a. $\frac{3}{2} = 1\frac{1}{2} \quad \frac{5}{3} = 1\frac{2}{3}$

$1\frac{1}{2} = 1\frac{2}{3}$

$+ 1\frac{2}{3} = 1\frac{4}{3}$

$2\frac{2}{3} = (3\frac{1}{3})$

11.) $240 \times 240 = 57,600$

$10.9 \times 10.9 = 118.81$

$.9 \times .9 = .81$

$57,600 + 118.81 - .81 = (57,718)$
 Bills

12.) $4. \frac{65+70+55+x}{4} = \frac{60 \cdot 4 = 240}{190+x=240} \quad (x=50)$

1.) 14
 $3\frac{1}{3}$ (12 qts)
 $+ 6\frac{2}{3}$ (3 gal.)
 $23\frac{1}{3} = 24 \text{ pts}$

2.) $7 = 6\frac{2}{3}$
 $- 2\frac{2}{3} = 2\frac{2}{3}$
 $(4\frac{2}{3})$

3.) $\begin{array}{r} A \quad B \quad C \\ *95.00 \quad 115 \quad 105 \\ \times .80 \text{ (20\% off)} \quad .90 \text{ (10\% off)} \\ \hline 92.00 \quad 94.50 \end{array}$

4.) $89^\circ + 1^\circ = 90^\circ$
 $89^\circ + 91^\circ = 180^\circ$

5.) a. $(7,283.276)$
 b. (10)

6.) $16 \times 16 = 256$
 $4 \times 4 = 16 \times 4 = 64 \times 4 = 256$
 $4(4)$

7.) 12.1
 5.2
 10.8
 3.0
 (31.1 cm)

8.) $n + -3\frac{1}{3} = 6\frac{2}{3} = 6\frac{4}{6}$
 $3\frac{1}{3} = 3\frac{2}{3} = 3\frac{4}{6} +$
 $n = 9\frac{4}{6} = 10\frac{2}{3}$

9.) (Math is fun.)

10.) $1000 \div 10 = 100 \times 100 = (10,000)$

11.) $(.002)$

12.) a. $(.70)$ b. $(.02)$

1.) $(-.03, -.003, .03, .3)$

2.)
$$\begin{array}{r} .77 \\ \times .07 \\ \hline 539 \\ 00 \\ \hline 0539 \end{array}$$

3.) $\frac{1}{4} = .25000$

$a = .25900$

$b = .26000$

$c = .24999$

4.) See answer sheet.

5.) $6+8=14$ (15)
 $\frac{4}{8} + \frac{3}{4} = 1$

6.) $\frac{18}{72} = \frac{1}{4}$

$\frac{12}{48} = \frac{15}{60} = \frac{18}{72} = \frac{24}{96} = \frac{21}{84}$

$48 \div 4 = 12 \quad 15 \times 4 = 60$

$96 \div 4 = 24 \quad 21 \times 4 = 84$

7.) Cross multiply:

$$\begin{array}{r} \times 200 \\ 6 \cancel{2} \times 4 \\ \hline 4x = 200 \times 6 \\ 4x = 1200 \\ 4x \div 4 = 1200 \div 4 \\ \hline x = 300 \end{array}$$

8.) Row 1 = 12 - 1 = 11 Row 5 = 12 - 2 = 10
 Row 2 = 12 ÷ 2 = 6 Row 6 = 12 × $\frac{3}{4}$ = 9
 Row 3 = 12 × $\frac{2}{3}$ = 8 Row 7 = 12 × $\frac{5}{6}$ = 10
 Row 4 = 12 × $\frac{3}{5}$ = 8 (62 dogs)

9.) $10\frac{1}{2}\% = (.105)$

10.) (300)

11.) a. $36 \overline{) 3.5} \quad (9.7)$

b. $48 \overline{) 14.9} \quad (10.2)$

12.) a. 4576.1093 b. 4576.1093

1.) $1 \text{ km} = 1000 \text{ m} \quad 1 \text{ m} = 1000 \text{ mm}$
 $1\frac{1}{2} \text{ km} \times \frac{1000 \text{ m}}{1 \text{ km}} = (14,500 \text{ m}) \times \frac{1000 \text{ mm}}{1 \text{ m}} = (14,500,000 \text{ mm})$

2.) a. $3\frac{1}{2} \times 7\frac{1}{3} = \frac{19}{2} \times \frac{22}{3} = \frac{418}{6} = 27\frac{13}{6}$

b. $3\frac{3}{10} \div \frac{1}{4} = \frac{33}{10} \div \frac{1}{4} = \frac{33}{10} \times \frac{4}{1} = \frac{132}{10} = 13\frac{2}{5}$

3.) a. $20 \overline{) 1.00} \quad (.05 = 5\%)$ b. $5 \overline{) 14.00} \quad (.80 = 80\%)$

4.) a. $\frac{40}{1000} = \frac{4}{100}$ (yes)

b. $400 \text{ h} = .400$
 $40 \text{ hun} = .40$ (yes)

5.) a. right angle = 90° $90 - 77 = 13^\circ$

b. obtuse = > 90
 so $77 \times x = 91$
 $x = 14^\circ$

6.) $9" \times 9" \times 9" = (729 \text{ cu. in.})$ 7.) a. $15 \overline{) 75.00} \quad (500)$ b. (500)

8.) $289 + -y = 386$
 $-289 \quad -289$

$-y = 97$
 $y = -97$

9.) $6.5 \text{ ft} = 6 \text{ ft. } 6 \text{ in.} = 5 \text{ ft } 18 \text{ in.}$
 $5.75 \text{ ft} = 5 \text{ ft. } 9 \text{ in.} = 5 \text{ ft. } 9 \text{ in.}$
 (9 in.)

10.) 1 mile = 5280
 $a. \frac{5280}{5} = (1056)$

b. $\frac{528}{100} = (52.8)$

11.) 7 hrs. 11 min. = 6 hrs. 71 min.
 $- 5 \text{ hrs. } 34 \text{ min.} = 5 \text{ hrs. } 34 \text{ min.}$
 (1 hr. 37 min.)

12.) $7\frac{1}{2}\% = .075$
 $.075 \times 1000 = (\$75.00)$