

## Geometry

- Identify parallel, perpendicular and intersecting lines.
- Find the area of a parallelogram and a triangle.
- Find the perimeter of a square given the length of one side.
- Determine the perimeter of a rectangle.
- Draw a figure with 4 congruent sides and 4 right angles.
- Identify right, acute and obtuse angles.
- Find the degree of an angle in a triangle, given the other two angles.
- Find the degree of an angle within a circle given the other angle.
- Find the circumference of a circle.
- Find ordered pairs on a grid.

## Data Analysis, Statistics and Probability

- Interpret information on a graph.
- Transfer information on a line graph to a bar graph.
- Predict the probability of heads or tails in the flip of a coin.

## Optional Supplemental Lessons

- Correctly identify and use math verbs in a number sentence.
- Evaluate expressions using the rules for the order of operations.
- Describe rate as a fraction of two numbers with different units.
- Describe ratio as a fraction of two numbers with the same units.
- Demonstrate an understanding of an opposite as an additive inverse.
- Demonstrate an understanding of the distributive, commutative and associative properties.
- Describe a pattern using variables.
- Use scientific notation to represent large and small numbers.
- Graph positive and negative rational numbers on a number line.
- Draw the perpendicular bisector of a line segment.
- Name and draw points on the xy-coordinates.
- Graph the solution to an equation on the xy-coordinates.
- Demonstrate an understanding of slope as a rate of change.
- Graph linear functions.
- Solve problems involving supplementary and complementary angles.
- Demonstrate an understanding of vertical and adjacent angles.
- Translate words to algebraic expressions or equations.
- Create and apply an algebraic equation to solve a real life situation.
- Identify the range, mean, median and mode of a data set.
- Determine whether a sample group may be biased.
- Use circumference and area formulas to evaluate problems involving circles.
- Accurately draw a geometric figure from a given set of conditions.

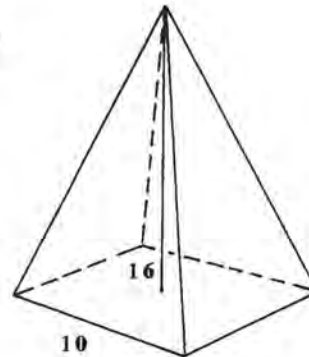
### Instructions for the User

- To use Summer Math Skills Sharpener, simply tear off a page and complete it. The program is designed to be used 3 days per week for 12 weeks. Please allow about an hour per page.
- Supplemental lessons have been added to address changes in state and National Standards. These lessons are OPTIONAL.
- Detailed solutions to all problems are included at the back of the book. Please complete an entire sheet prior to checking your answers.
- All concepts are part of a standard geometry curriculum. Please attempt all problems. In addition to the solutions, pink “Help Pages” have been included to assist you in completing the problems.
- A yellow “Glossary of Terms” is located at the back of the book.
- A green list of “Commonly used Postulates and Theorems” is included to assist you with your proofs.
- Pages should be worked in order. While each page contains mixed concepts, individual concepts, within the book, have been ordered from easier to more difficult.
- If you experience difficulty with certain concepts, address the problems with your teacher. He or she may recommend additional help in these areas.
- It is important to give every problem your best effort. Problems may seem challenging, but use a combination of the “Help Pages” and the “Solutions” to assist you for maximum success.
- This book can be used to supplement ACT and other placement test reviews. Among the problems are concepts that appear regularly on these tests.
- We appreciate your comments. Please complete the enclosed evaluation page after you have entered your next math course but before November 1st.

1.  $\triangle ABC$  is isosceles with base  $BC$ . The measure of  $\angle A$  is four times the measure of  $\angle B$ . Find the measure of  $\angle C$ .

2.  $B$  is the midpoint of  $\overline{AC}$ . If  $AB = 3x + 5$  and  $BC = 17 - 5x$ , find  $AC$ .

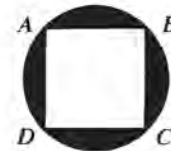
3. Given the regular square pyramid pictured at the right,  
a. find the surface area.



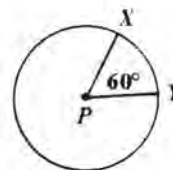
b. find the volume.

4. One angle of an isosceles triangle measures  $42^\circ$ . What are the possible measures for the other angles of this triangle?

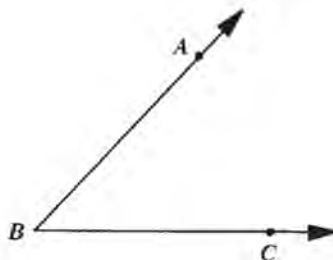
5. Find the area of the shaded region in the figure at the right if  $ABCD$  is a square inscribed in circle  $\odot$  and  $AC = 6\sqrt{2}$ .



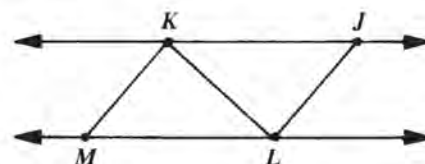
6.  $\odot P$  has radius,  $r = 5$ . Find  $m\widehat{XY}$ .



7. Find  $m\angle ABC$ .



8. Given:  $\overrightarrow{KJ} \parallel \overrightarrow{ML}$ ,  $\overline{KJ} \cong \overline{LM}$   
Prove:  $\triangle KJL \cong \triangle LMK$

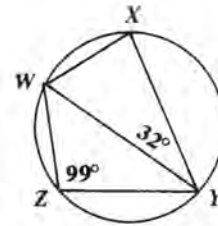


(Write your proof on the back of page 16.)

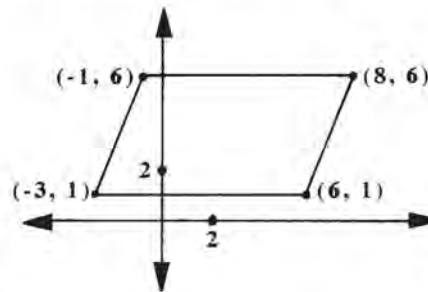
1. Consider the following statements.
  - a. Every labrador retriever is a dog.
  - b. Zeus is a chocolate labrador retriever.
 What, if anything, can you conclude from these two statements?

2. Two similar antique vases are one foot and two feet tall. If it takes one pint of paint to restore the larger vase, how much paint will the smaller vase require?

3. Use the circle at the right to find
  - a.  $m\widehat{WXY}$  \_\_\_\_\_
  - b.  $m\widehat{XY}$  \_\_\_\_\_
  - c.  $m\angle WXY$  \_\_\_\_\_
  - d.  $m\widehat{WZY}$  \_\_\_\_\_



4. A parallelogram has vertices  $(8, 6)$ ,  $(6, 1)$ ,  $(-3, 1)$ , and  $(-1, 6)$ . Find each of the following:
  - a. the perimeter of the parallelogram.



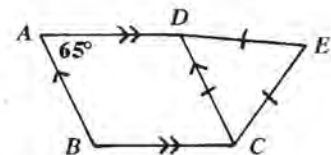
- b. the area of the parallelogram.

5. WOOD radio services customers within a 50 mile radius of its broadcasting tower. How large is the area that the station services?

6. The angles of a triangle are in a ratio of 1:2:6. Find the measure of each angle.

7. Susie Fowlup has done it again. She claims that  $\triangle ABC \cong \triangle DEF$  because  $AB = DE = 2$  inches,  $m\angle A = m\angle D = 35^\circ$ , and  $m\angle B = m\angle F = 55^\circ$ . On the back of page 28, accurately draw and label both triangles to show her the error.

8. The figure at the right is made from a parallelogram  $ABCD$ , and equilateral triangle  $\triangle CDE$ . Find the  $m\angle ADE$ .

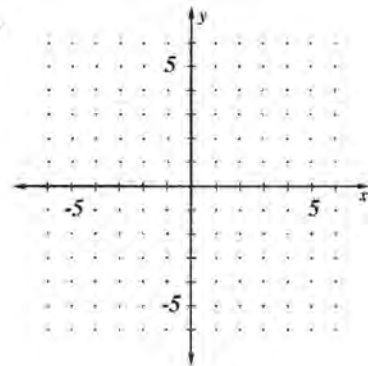


1. a. Use the equation  $x^2 + y^2 = 25$  and complete the table.

Remember to include the plus and minus values.

$x =$     -5    -4    -3    0    3    4    5

$y =$



- b. Graph all points on the coordinates at the right and connect with a smooth curve.
- c. Identify this geometric figure. \_\_\_\_\_

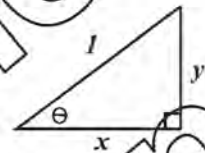
2. From eye level five feet above the ground, a person has to look up at an angle of  $40^\circ$  to the top of a flagpole 45 feet away. (Measure distance along the horizon.)

- a. Draw and accurately label a sketch of this.
- b. Find the height of the flagpole.

3. Use the right triangle pictured at the right.

Find the following:

- a.  $\sin\theta =$                       b.  $\cos\theta =$                       c.  $\tan\theta =$



Use this information to prove two identities.

- d. Since  $y = \sin\theta$  and  $x = \cos\theta$ , by substitution  $\tan\theta =$  \_\_\_\_\_.

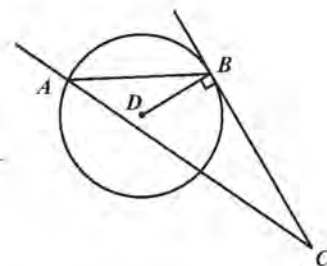
- e. By the Pythagorean Theorem you know  $x^2 + y^2 = 1^2$  or  $x^2 + y^2 = 1$ .

Substitute for  $x$  and  $y$  to find the Pythagorean Identity: \_\_\_\_\_.

Note  $\sin^2\theta = (\sin\theta)^2$ .

4. Use the figure pictured at the right. Identify the following lines:

- a. tangent \_\_\_\_\_      b. chord \_\_\_\_\_      c. secant \_\_\_\_\_



**Point, line plane postulate:**

- a. Two points determine exactly one line.
- b. Every line consists of points that can be put into a one-to-one relationship with the real numbers.
- c. A point has no dimension.  
A line has one dimension.  
A plane has two dimensions.
- d. Two different lines intersect in at most one point.

**Triangle inequality postulate:** The sum of the lengths of two sides of a triangle is greater than the length of the third side.

**Postulates of logic:**

- a. Law of detachment: Given the true statement  $p \Rightarrow q$ , then given  $p$  one can conclude  $q$ .
- b. Law of transitivity: Given the true statements  $p \Rightarrow q$  and  $q \Rightarrow r$ , then given  $p$  one can conclude  $r$ .
- c. Law of the contrapositive: Given the true statement  $p \Rightarrow q$ , one can conclude not  $-q \Rightarrow$  not  $-p$ .
- d. Law of ruling out possibilities: If  $p$  or  $q$  is true and  $p$  is false, then  $q$  is true.
- e. Law of indirect reasoning: If  $p$  or not- $p$  is true, and  $p$  is proven false, then not- $p$  is true.

**Linear pair theorem:** If two angles form a linear pair, then they are supplementary.

**Vertical angles theorem:** If two angles are vertical angles, then they are equal.

**Parallel lines and slopes theorem:** If two lines are parallel, then they have the same slope.

**Perpendicular lines and slopes theorem:** Two nonvertical lines are perpendicular if the product of their slopes is  $-1$ .

**Transitivity of parallelism:** If two lines are parallel to the same line, then they are parallel to each other.

**Perpendicular to parallels theorem:** If a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.

**CPCF theorem:** If two figures are congruent, then their corresponding parts are congruent.

**A-B-C-D theorem:** Every isometry preserves angle measure, betweenness, collinearity, and distance.

**Reflexive property of congruence:** Every figure is congruent to itself.

**Transitive property of congruence:** If  $A \cong B$  and  $A \cong C$  then  $B \cong C$ . Or, two figures congruent to the same figure are congruent to each other.

**|| lines  $\Rightarrow$  AIA  $\cong$ :** If two parallel lines are intersected by a transversal, then the alternate interior angles are equal.

**acute angle:** Angle measuring greater than  $0^\circ$  and less than  $90^\circ$ .

**adjacent angle:** Two nonstraight and nonzero angles with a common side interior to the angles formed by the uncommon sides.

**algebraic equation:** A math sentence relating two expressions as equal.

**altitude:** The perpendicular distance from the vertex of a triangle to the side opposite. Also, the perpendicular distance between parallel lines.

**angle:** The union of two rays, the sides, at a point, the vertex.

**angle bisector:** A ray that divides an angle into two equal parts.

**angle of depression:** The angle measured from the horizontal downward from the observer's eye.

**area:** The number of unit squares or parts of unit squares required to tile a plane figure.

ex. parallelogram:  $A = bh$

rectangle:  $A = lw$

triangle:  $A = \frac{1}{2}bh$

trapezoid:  $A = \frac{1}{2}h(b_1 + b_2)$

circle:  $A = \pi r^2$

**base:** The variable  $b$  in the expression  $b^n$ .

**bisector:** A point, line, ray, or plane that divides a segment, angle, or figure into two parts of equal measure.

**circle:** The set of all points, the radius, equal distance from a certain point, the center.

**circumference:** The perimeter of a circle.  $c = \pi d$ . The ratio of the circumference to the diameter is  $\pi$ .

**collinear points:** Points that line on the same line.

**complementary angles:** Two angles whose sum is  $90^\circ$ .

**cone:** The surface of a conic solid whose base is a circle.

**congruent figures:** Two figures,  $A$  and  $B$ , are congruent if one is the image of the other under a reflection or a composite of reflections.

**conic solid:** The set of points between a given point, the vertex, and the points in a given region, the base, and the vertex and the base.

**construction:** A drawing made using a straight edge and compass and following a set of rules.

**converse:** A statement formed by switching the antecedent and the consequent of a given statement.

**coordinates:** An ordered pair.

**PYTHAGOREAN THEOREM:**  $\{c^2 = a^2 + b^2\}$  The Pythagorean theorem states that, in a Right Triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides (called legs). Whenever you have two of the three lengths, then you can find the third.

example: Given  $a = 4$ ,  $b = 3$ , find the hypotenuse.

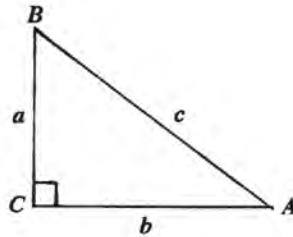
$$c^2 = a^2 + b^2$$

$$c^2 = 4^2 + 3^2$$

$$c^2 = 16 + 9$$

$$c^2 = 25$$

$$c = 5$$



**DISTANCE FORMULA:** Distance is the absolute distance between two points. In the coordinate system, it can be found using:  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ .

Notice that the distance formula is very closely related to the Pythagorean theorem.

$$c^2 = a^2 + b^2$$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2, \quad \text{so} \quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

example: Find the distance between  $(-3, 5)$  and  $(5, -6)$ .

$$d = \sqrt{(5 - (-3))^2 + (-6 - 5)^2}$$

$$d = \sqrt{8^2 + (-11)^2}$$

$$d = \sqrt{185}$$

$$d \approx 13.6$$

**SLOPE:** Slope is the rate of change. It is represented by the variable  $m$ .

On a graph  $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}}$

example: Find the slope of the line through  $(-3, 5)$  and  $(5, -6)$ .

$$m = \frac{-6 - 5}{5 - (-3)}$$


$$m = \frac{-11}{8}$$

For every horizontal move of 1 unit to the right, there is a vertical change of  $-\frac{11}{8}$ .

Taking this one step further, when an equation is written in the slope/intercept form,  $y = mx + b$ , the  $m$ , which is the coefficient of the  $x$ , is the slope.

# SOLUTIONS

pg. 1

① zero, one, two      ② a) 

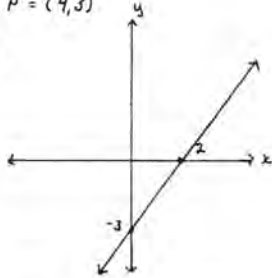
③  $-15+8=-7$  or  $-15-8=-23$       b) ray

④  $C=\pi \cdot 2r = 20\pi$  cm.      ⑤  $15.6 \cdot 8.4 = 7.2$

⑥  $5x+20=2x+56$        $5(12)+20=80$   
 $3x=36$        $m\angle 2 = m\angle 4 = 80^\circ$   
 $x=12$        $m\angle 1 = m\angle 3 = 100^\circ$

⑦  $P' = (4, 3)$

⑧



⑨  $x^2+5x+2x+10$   
 $x^2+7x+10$

⑩ a)  $S \cap T = \{\overline{AB} \cup \overline{BC}\}$       b)  $S \cup T = \{\overline{ABCD} \cup \overline{AC}\}$

pg. 2

① a)  $\{2, 4\}$       b)  $\{1, 2, 3, 4, 5, 6, 8, 10\}$

② a)       b) Intersection is a point  $(2, 4)$

③ No  $14+15 < 30$  By the triangle inequality theorem, the sum of two sides of a triangle must be greater than the third side.

④  $k = \frac{1}{4}$        $k^2 = \frac{1}{16}$  the weight       $k^2 = \frac{1}{16}$  the area

⑤ Ingredients is the area, so if  $k = \frac{16}{10} = \frac{8}{5}$ , then  $k^2 = \frac{64}{25}$       ⑥  $-x+60+6x+5=180$   
 $5x+65=180$   
 $5x=115$   
 $x=23$   
 $-23+60=37^\circ$   
 $6(23)+5=143^\circ$

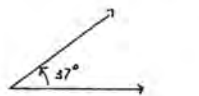
⑦  $A' = (3, 1)$        $B' = (-2, 4)$        $C' = (0, -4)$

⑧ a)  $\frac{3}{5} = \frac{x}{230}$       b)  $5x = 690$   
 $x = 138$  girls

pg. 3

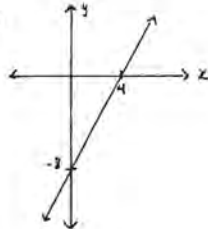
①  $m\angle 3 = m\angle 1 = 57^\circ$       Vertical angles are equal.  
 $m\angle 4 = m\angle 2 = 123^\circ$       If two angles form a linear pair then they are supplementary.

②



③  $0 < 23+x < 90$   
 $-23 < x < 67$

④



a)  $(1, 0)$       b)  $(0, -8)$

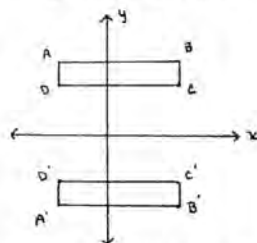
⑤ Let  $x =$  smaller angle  
 $3x =$  larger angle  
 $x+3x=90$   
 $4x=90$   
 $x=22.5^\circ$        $3x=67.5^\circ$

⑥  $A = \pi r^2$   
 $A = \pi \cdot 4^2 = 16\pi$  cm<sup>2</sup>

⑦  $12\text{ft} = 144$  in.       $16\text{ft} = 192$  in  
 $A = 144 \times 192 = 27648$  in<sup>2</sup>  
 $A_{\text{brick}} = 32$  in<sup>2</sup>       $\frac{27648}{32} = 864$  bricks

⑧  $3^2 = 9$  times

a)  $A' = (-2, -3)$        $B' = (3, -3)$        $C' = (3, -2)$        $D' = (-2, -2)$   
 c) rectangle  
 d)  $A = lw = 5 \cdot 1 = 5$  sq. units



pg. 4

① parallel      ② parallel

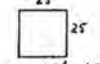
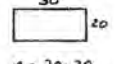
③  $.8+.9=1.7$        $1.7 > 1$       Yes. The sum of any two sides is greater than the 3rd side.  
 $.8+1=1.8$        $1.8 > .9$   
 $.9+1=1.9$        $1.9 > .8$

④  $|-31-42| = 73$

⑤  $\frac{-5}{1000} = \frac{1}{2000} = \frac{\text{model}}{\text{bridge}}$  or  
 $\frac{2000}{1} = \frac{\text{bridge}}{\text{model}}$

⑥  $8y-23=4y+41$   
 $4y=64$   
 $y=16$

⑦ square  $\frac{100}{4} = 25$       rectangle (ex.)

  $A = 25^2 = 625$  ft<sup>2</sup>        $A = 20 \cdot 30 = 600$  ft<sup>2</sup>

⑧  $m = \frac{3-2}{5-(-3)} = \frac{1}{8}$

circle  $100 = \pi d$   
 $d \approx 31.8$   
 $r \approx 15.9$   
 $A = \pi r^2 = \pi \cdot 15.9^2 \approx 794$  ft<sup>2</sup>

Conclusions	Justifications
① $m\angle 1 = m\angle 2$	① Given
② $m\angle 1 = m\angle 3$	② Vertical angles are =.
③ $m\angle 2 = m\angle 3$	③ Transitive property of equality