ALGEBRA

The student will

compare and order real numbers. (rational and irrational)

demonstrate an understanding of the exact and the approximation for an answer.

demonstrate an understanding of reciprocals and opposites.

demonstrate an understanding of the concept of rate and use it in real life problems.

write numbers in scientific and in decimal notation.

determine the absolute value of numbers.

write an algebraic sentence using a math verb. ($>$, $<$, $\geq$, $\leq$, and $=$)

find the square root of a number both as an exact value and as an approximation.

simplify radical expressions.

use the order of operations to simplify algebraic expressions and algebraic number sentences.

find the product of positive and negative numbers.

demonstrate an understanding between a table, an equation, and a graph.

describe a ratio as a fraction of two numbers with the same units.

find decimal and percent equivalents to simple fractions.

use the properties of exponents (power rules) to simplify algebraic expressions.

solve equations and inequalities using absolute values.

use the multiplicative and additive properties to solve equations and inequalities.

solve an inequality and graph the solution on a number line.

solve an absolute value inequality and graph the solution on a number line.

multiply polynomials and simplify the final answer. (FOIL binomials)

add, subtract, multiply and divide rational expressions.

generalize a pattern or a table using an algebraic expression or formula.

find the prime factorization of a monomial.

find the degree, the leading coefficient, the number of terms, and the constant for a polynomial,

factor certain trinomials and binomials.

divide polynomials.

demonstrate an understanding functions (linear, quadratic, and exponential) and function notation.

determine the domain and the range of a function.

perform a two dimensional slide on the xy-coordinate system.

find the slope of a line using two points on the line or an equation for the line.

find the distance between two points.

find the midpoint of two points.

demonstrate an understanding of slope and its relationship to parallel and perpendicular lines.

describe the slope of horizontal and vertical lines.

graph lines using the slope and y-intercept.

graph lines using the x- and y- intercepts.

graph quadratic equations.

graph a system involving an inequality on the xy-coordinates.

graph a system of equations on the xy-coordinates and find the solution set.

write and solve proportions.

select and use formulas to solve problems.

demonstrate an understanding of and use the associative, commutative, and distributive properties to simplify expressions and to solve equations.

use the concepts of equations and graphs to solve real life equations.

solve problems involving percent.

use equations and graphs to represent exponential growth and decay.

use quadratic equations to explore projectiles.

create and solve an equation created from a data set.

apply algebraic techniques to solve rate, work, and percent mixture problems.

find the next term, the constant difference, and a rule for an arithmetic sequence.

find the next term, the constant rate of change, and a rule for a geometric sequence.

identify the elements, the domain, the measures of central tendency, and the range of a data set.

calculate probabilities for simple events.

demonstrate an understanding of factorials.

calculate the probability for a geometric region.

use a graphics calculator to graph an equation.

find the appropriate window on a graphics calculator in order to visualize an entire graph.

use various features found on a graphics calculator to answer questions about a graph.

explore various functions and editing features on a graphics calculator.

find an equation for a line given two points or a point and the slope.

solve a system of simultaneous equations using graphing or using algebraic solutions. (substitution and addition methods)

determine the slope-intercept or standard form of an equation.

solve equations involving exponents or square roots.

find the solutions to a quadratic equation using factoring, the Quadratic Formula, graphing, and completing the square.

find the number of real solutions to a quadratic equation using the discriminant.

find the vertex of a parabola and demonstrate an understanding of minimum and maximum.

demonstrate an understanding of the Pythagorean Theorem.

demonstrate an understanding of the Triangle Inequality Postulate.

explain whether a conclusion is made using deduction or induction.

Algebra 1

Instructions for the User

- **To use Summer Math Skills Sharpener, simply tear off a page and complete it. The program is designed to be used 3 days per week for 10 weeks.**
- **Supplemental lessons have been included to address changing state and National Standards. These lessons are OPTIONAL. (See green supplement following body of program.)**
- **Detailed solutions to all the problems are included at the back of the book. Please complete an entire sheet prior to checking your answers. You may want to remove the solutions and store them in a place apart from the book.**
- **All concepts are part of a standard algebra 1 curriculum. Please attempt all problems. In addition to the solutions, pink "Help Pages" have been included to assist you in completing the problems.**
- **A yellow "Glossary of Terms" is located at the back of the book.**
- **Pages should be worked in order. While each page contains mixed concepts, individual concepts, within the book, have been ordered from easier to more difficult.**
- **If you experience difficulty with certain concepts, address the problem with your teacher. He or she may recommend additional help in these areas.**
- **It is important to give every problem your best effort. Problems may seem challenging, but use a combination of the "Help Pages" and the "Solutions" to assist you for maximum success.**
- **We appreciate your comments. Please complete the enclosed evaluation page after you have entered your next math course but before November 1st.**

For problems 1 – 3, solve for x.

1. $x - 7 - 4x = -4$
2. $2(x-1) - 7 = 5$
3. $7 - 4(x+1) = -1$

4. **The perimeter of a rectangular lot is 156 meters. The length is twice the width.**

   a. **Write the equation that describes this situation.**

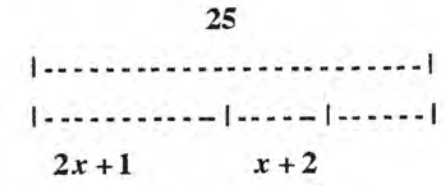
   b. **Find the length and the width.**

5. **The perimeter of an equilateral triangle is 7.5 inches. What is the length of one side?**

6. **Evaluate $3(k-j)$ where $k = \frac{1}{3}$ and $j = \frac{4}{9}$.**

7. **(8, 3) and (5, y) lie on the same line. If the slope of the line is $\frac{1}{2}$, find y.**

8. **Write the inequality to represent the diagram.**



9. **Graph $x \geq -2$.**

10. Multiply each of the following.

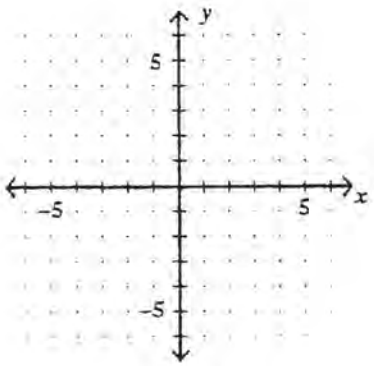
a. $(x-1)(2x^2+x+5)$

b. $(x+3)(x-3)(2x+1)$

11. Find a number between $\frac{9}{16}$ and $\frac{21}{32}$.

12. A euchre deck of cards consists of 24 cards. It includes the 9 through ace of each suit. A hand consists of 5 cards. In a particular hand, what is the probability of being dealt the jack of spades and the jack of clubs?

13. a. Find three pairs of numbers that satisfy the equation $y = 2x - 3$.



b. Plot these points on the graph at the right and connect them with a line.

c. When an equation is in the form $y = mx + b$, m represents the slope of the line. What is the slope of this line?

d. Choose two of your points and verify this.

For problems 1 – 4, solve for x.

1. $\sqrt{x} = 25$ 2. $x^2 = 25$ 3. $|x| = 25$ 4. $|x+1| = 25$

5. **In 2000 Lincoln High School had 804 students. It has been gaining about 30 new students each year. In that same year Washington High School had 1008 students. It has been gaining only about 10 new students each year.**

   a. **Write an equation that states after x years both schools will have the same number of students.**

   b. **Solve for x. In what year will the schools have the same number of students?**

6. **Factor each of the following.**

   a. $x^2 + x - 6$ b. $x^2 - 49$ c. $x^2 + 26x + 25$

7. **There are twelve players on the varsity basketball team. How many different ways can they line up for a team picture?**

8. **A slope of –3 means that for every horizontal change of 1, there is a vertical change of _____.**

9. The circumference of a circle (C) is given by the formula $C = 2\pi r$ or $C = \pi d$ where r is the radius and d is the diameter.

   a. Find the exact circumference of a circle with $d = 8$ units.

   b. Approximate the circumference to the nearest tenth.

   c. Find the exact area of the same circle.

   d. Approximate the area to the nearest tenth.

10. The equation $ax^2 + bx + c = 0$ can be solved for x using the *quadratic formula* or $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The *discriminant* D is $b^2 - 4ac$. Always calculate this first. Then you can solve $x = \dfrac{-b \pm \sqrt{D}}{2a}$.

    Solve $x^2 + 7x + 12 = 0$

    a. Find the discriminant.

    b. Find x.

11. a. Find the slope of the line determined by $(-3, 3)$ and $(-4, -1)$.

    b. Using the language of problem 8, state what this means.

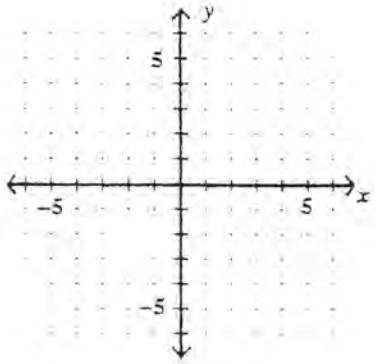
For problems 1 – 3, simplify.

1. $\dfrac{\frac{2}{3}}{\frac{1}{6}}$

2. $\dfrac{\frac{2}{x}}{\frac{3}{x^2}}$ $x \neq 0$

3. $(x^3)^{-2} \bullet (x^2)^3$ $x \neq 0$

4. a. **Solve by graphing.** $\begin{cases} y = -2x - 1 \\ y = \frac{1}{2}x + 1 \end{cases}$



   b. **What are the coordinates of the solution set?**

   c. **Two lines are perpendicular when the product of their slopes is –1. Verify that these lines are perpendicular.**

5. **A perfectly square lot has a perimeter of exactly 1 mile.**

   a. **Find the length of one side.**

   b. **Find the area.**

6. **Solve for x.**

   a. $(x-3)^2 = 36$

   b. $x^2 - 10x + 25 = 64$

7. The points $(2, 4)$ and $(-2, 8)$ lie on a line.

   a. Find the slope of this line.

   b. Find the y-intercept of this line.

   c. Write an equation for this line.

For problems 8 – 9, use any method to solve for x and y.

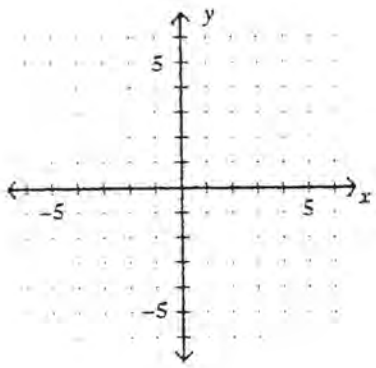
8. $\begin{cases} 3x + 8y = 2 \\ -3x - 2y = -5 \end{cases}$

9. $\begin{cases} x = 2y \\ 2x + y = 55 \end{cases}$

10. a. Make a table of values and graph $y = x^2 + 2x - 3 = 0$

    b. Find the x-intercepts.

    c. Find the y-intercept.



    d. $\frac{-b}{2a}$ is a formula that gives the x coordinate of the vertex. Substituting this value into the function gives the y-coordinate. Use this method to find the vertex.

For problems 1-3, simplify.

1. $(x-5)(2x+8)$ **2.** $(x^2-2x+3)(x+2)$ **3.** $(x^2-3x+9)(x+3)$

For problems 4-5, simplify by first writing as a complex fraction. Recall that "*per*" means division. Label with the correct units.

Example:

$$\frac{\$3.36 \text{ per dozen}}{12 \text{ cookies per dozen}} = \frac{\frac{\$3.36}{1 \text{ dozen}}}{\frac{12 \text{ cookies}}{1 \text{ dozen}}} = \frac{\$3.36}{1 \text{ dozen}} \cdot \frac{1 \text{ dozen}}{12 \text{ cookies}} = \$.28 \text{ per cookie}$$

4. $\dfrac{8000 \text{ miles per hour}}{200 \text{ gallons per hour}}$ **5.** $\dfrac{150 \text{ heartbeats per min}}{0.25 \text{ miles per min}}$

6. If the area of a rectangle is 24 square units, its perimeter is 20 units.

a. Draw and label an accurate instance of this conditional.

b. Draw and label a counter example to this conditional.

7. Provide the justifications for each step in the simplification below.

$2(a+3)+4(a-3)$	Given
$2a+6+4a-12$	_____
$2a+4a+6-12$	_____
$6a-6$	_____

8. The sum of the angles of a triangle is 180°. Figure *ABC* is a triangle. You conclude that the sum of the angles is 180°.

a. Is this an example of deductive or inductive reasoning?

b. Justify your answer.

9. a. You mix 20 chocolate covered raisins with 35 peanuts. What percent of your mix is chocolate covered raisins?

   b. A box contains 26 pens and pencils. If $x\%$ of the mix is pencils, how many pencils are in the box?

   c. You mix x orange jelly beans with 25 black jelly beans. What percent of your mix is orange jelly beans?

10. Each fall Justin and Tyrone rake leaves to earn extra money. Justin can rake an average yard in 5 hours. Tyrone can rake an average yard in 6.5 hours. How long will it take them to rake a yard if they work together?

    a. What part of a yard can Justin rake in one hour?

    b. What part of a yard can Tyrone rake in one hour?

    c. What part of a yard can they rake in one hour if they work together? (This is the sum of your answers to a and b.)

    d. Let t = the time it takes to do the job together. In terms of t what part of the yard can they complete in one hour?

    e. Using your answers to "c" and "d", find how long it will take them to rake the yard if they work together.

11. Find an equation for the line parallel to $y = -2x + 4$ containing $(2, 5)$.

acute angle : an angle measuring greater than 0 degrees and less than 90 degrees.

algebraic equation : a math sentence relating two expressions as equal.

algebraic expression : a combination of numbers and variables.

altitude : the perpendicular distance from the vertex of a triangle to the side opposite. Also, the perpendicular distance between parallel lines.

angle : the union of two rays (the sides) at a point (the vertex).

area : the number of unit squares or parts of unit squares required to tile a plane figure.

ex. parallelogram : $A = hb$

rectangle : $A = hb$

triangle : $A = \frac{1}{2}hb$

trapezoid : $A = \frac{1}{2}h(b_1 + b_2)$

circle : $A = \pi r^2$

arithmetic sequence : sequence with a constant difference.

base : the variable b in the expression b^n.

binomial : a polynomial containing two terms.

circle : the set of all points (the radius), equal distance from a certain point (the center).

circumference : the perimeter of a circle. $c = \pi d$. The ratio of the circumference to the diameter is π.

complementary angles : two angles whose sum is 90°.

composite of functions : an operation that first applies one function, then the other. It is written $f(g(x))$ or $f \circ g(x)$. Both mean f following g of x.

coordinates : an ordered pair.

ORDER OF OPERATIONS: Parentheses, powers, multiplication, division, addition, subtraction.

{use "Pretty Please, My Dear, Aunt Sally" to help remember}

ex. $(x-2x)^2+2x^2-3(x+3x)$

1. Parentheses first : $(-x)^2+2x^2-3(4x)$

2. Powers second : $x^2+2x^2-3(4x)$

3. Multiplication and/or Division third : x^2+2x^2-12x

4. Addition and/or Subtraction of LIKE terms last : $3x^2-12x$

PYTHAGOREAN THEOREM : $\{c^2=a^2+b^2\}$ **The Pythagorean theorem states that, in a RIGHT triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides (called legs). Understanding this is essential to a complete understanding of right triangles, similar triangles, and trigonometry. Whenever you have two of the three lengths, then you can find the third.**

ex. Given hypotenuse $c=13$, **leg** $b=5$, **find leg** a.

$c^2=a^2+b^2$

$13^2=a^2+5^2$

$169=a^2+25$

$144=a^2$

$a=12$

SLOPE : Slope is the rate of change. It is represented by the variable m.

On a graph $m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{change\ in\ y}{change\ in\ x}=\dfrac{rise}{run}$

A popular description is to think of slope as the amount of vertical change for every horizontal move of 1 unit to the right.

ex. Find the slope of the line through (1, - 1) and (3, 5)

$m=\dfrac{y_2-y_1}{x_2-x_1}$

$m=\dfrac{5-(-1)}{3-1}$

$m=\dfrac{6}{2}=3$

For every horizontal move of 1 unit to the right, there is a vertical change of + 3.

Taking this one step further, when an equation is in the slope/intercept form, $y=mx+b$, **the** m, **which is the coefficient of the** x, **is the slope.**

Lesson 1 pg. 1

① $0 = x + (-13)$
$+13 \quad +13$
$13 = x$

② $\frac{1}{3}x = 12$
$\frac{3}{1}\cdot\frac{1}{3}x = 12\cdot\frac{3}{1}$
$x = 36$

③ $2x - 7 + 7 = 2x$
$2x = 2x$
$\frac{1}{2}\cdot 2x = 2x\cdot\frac{1}{2}$
$x = x$
$x =$ any real number

④ $\frac{2}{3}x = -\frac{4}{3}$
$\frac{3}{2}\cdot\frac{2}{3}x = -\frac{4}{3}\cdot\frac{3}{2}$
$x = -2$

⑤ $(1-1)(1-5) = 0 \quad 0\cdot -4 = 0$ T
$(3-1)(3-5) = 0 \quad 2\cdot -2 \neq 0$ F
$(5-1)(5-5) = 0 \quad 4\cdot 0 = 0$ T
$(7-1)(7-5) = 0 \quad 6\cdot 2 \neq 0$ F
$(9-1)(9-5) = 0 \quad 7\cdot 4 \neq 0$ F
$x = 1$ or $x = 5$

⑥ a. $3(x+1)+5$
$3x+3+5$
$3x+8$

b. $(x+2)(x+5)$
$x^2+7x+10$

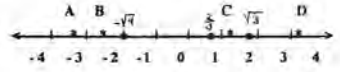
c. $-2(-3)$
6

⑦ a. I b. $.25 = \frac{1}{4}$ R c. $.\overline{3} = \frac{1}{3}$ R
d. $\frac{0}{1}$ R e. I f. $\sqrt{25} = \frac{5}{1}$ R

⑧ $20\cdot 1 = 20 \quad 20+1 \neq 12$
$10\cdot 2 = 20 \quad 10+2 = 12$
10 and 2

Lesson 1 continued pg. 2

⑨ a. C b. A c. D

⑩ 

⑪ a. 50 b. $\frac{10}{50} = \frac{1}{5}$ c. $\frac{50}{50} = 1$ d. $\frac{0}{50} = 0$

⑫ 

⑬ a. 5 b. doesn't exist c. 8
d. 8 e. -8 f. -8

⑭ Rename each fraction with a common denominator.
$\frac{1}{4} = \frac{3}{12} \quad \frac{1}{3} = \frac{4}{12}$
Since there is no integer between 3 and 4, rename again with a larger denominator.
$\frac{3}{12} = \frac{6}{24} \quad \frac{4}{12} = \frac{8}{24}$
Since 7 is between 6 and 8, $\frac{7}{24}$ is a number between $\frac{1}{4}$ and $\frac{1}{3}$
This is not the only solution. For example,
$\frac{3}{12} = \frac{9}{36} \quad \frac{4}{12} = \frac{12}{36}$
$\frac{10}{36}$ and $\frac{11}{36}$ are additional solutions.

Lesson 2 pg. 3

① $.5x = 3.5$
$\frac{.5x}{.5} = \frac{3.5}{.5}$
$x = 7$

② $\frac{x}{2} = -\frac{7}{4}$
$2\cdot\frac{x}{2} = -\frac{7}{4}\cdot 2$
$x = \frac{-7}{2}$ or -3.5

③ $\frac{2}{3}x + 309 = 711$
$\quad -309 \quad -309$
$\frac{2}{3}x = 402$
$\frac{3}{2}\cdot\frac{2}{3}x = 402\cdot\frac{3}{2}$
$x = 603$

④ $2(0)-5 = -5$
$2(1)-5 = -3$
$2(2)-5 = -1$
$2(3)-5 = 1$

⑤ $\frac{3(5+4)}{5(-3)-5} = \frac{3\cdot 9}{-15-5} = \frac{27}{-20} = -\frac{27}{20}$

⑥ a. $2x + 67 = 113$

b. $2x + 67 = 113$
$\quad -67 \quad -67$
$\frac{1}{2}\cdot 2x = 46\cdot\frac{1}{2}$
$x = 23$

⑦ $x + 4x = 25$

⑧ 

Lesson 2 continued pg. 4

⑨ a. $d = rt$
$d = 4\cdot 60 = 240$ mi.

b. $d = rt$
$d = (x+3)50 =$
$50x + 150$ mi.

⑩ a. $(x-3)(x+8)$
$x^2+5x-24$

b. $(x-5)(x-4)$
$x^2-9x+20$

c. $(x+4)^2$
$(x+4)(x+4)$
$x^2+8x+16$

⑪ a. 11 chips (outcomes) This is the sample space.
3 unique colors.
b. $\frac{5}{11}$ c. $\frac{4}{11}$ d. $\frac{0}{11} = 0$
e. $\frac{4}{11} + \frac{2}{11} = \frac{6}{11}$ f. $\frac{5}{11} + \frac{4}{11} + \frac{2}{11} = \frac{11}{11} = 1$
g. $\frac{4}{11}\cdot\frac{2}{10} + \frac{2}{11}\cdot\frac{4}{10} = \frac{16}{110} = \frac{8}{55}$
green then white or white then green

⑫ $-12\cdot 1 = -12 \quad -12+1 \neq -1$
$-6\cdot 2 = -12 \quad -6+2 \neq -1$
$-4\cdot 3 = -12 \quad -4+3 = -1$
-4 and 3

⑬ $\sqrt{\frac{1}{4}} = \frac{1}{2}$ Rational
$\frac{1}{2}$ is the ratio of two integers.