PREALGEBRA

The student will

compare and order real numbers. (rational and irrational)

demonstrate an understanding of place value.

name the base and exponent when given a number raised to a power.

recognize the math verbs and be able to use them correctly in a math sentence. ($>$, $<$, $\geq$, $\leq$, and $=$)

demonstrate an understanding of terminating and non terminating decimals.

find decimal and percent equivalents to simple fractions.

find different representations for the same fraction.

explore the powers of 10, both positive and negative, by moving the decimal place accordingly.

evaluate expressions using the rules for order of operations.

use the property of reciprocals to divide fractions.

describe rate as a fraction of two numbers with different units.

describe ratio as a fraction of two numbers with the same units.

write proportions and solve them using means-extremes.

demonstrate an understanding of the distributive property.

demonstrate an understanding of an opposite as an additive inverse.

demonstrate an understanding of the commutative and associative properties for addition and multiplication.

demonstrate an understanding of rounding and estimating.

convert length, mass, and capacity within the U.S. system and within the metric system.

select the appropriate unit of measure for a given situation.

measure and draw angles using a protractor.

use scientific notation to represent large and small numbers.

find the least common multiple and the greatest common factor for a set of numbers.

demonstrate an understanding that variables stand for numbers.

describe a pattern using variables.

demonstrate an understanding of function notation.

graph positive and negative rational and irrational numbers on a number line.

classify angles as acute, obtuse, right, and straight.

construct the perpendicular bisector of a line segment.

graph the solution to an inequality on a number line.

find the measure of the third angle of a triangle given the other two.

name and draw points on the *xy*-coordinates.

graph the solution to an equation on the *xy*-coordinates.

demonstrate an understanding of slope as a rate of change.

calculate the slope given two points on a line or the equation for a line.

graph linear, quadratic, and cubic functions.

draw the image of a polygon under a translation or a reflection.

draw a net for a prism or a pyramid.

solve problems involving supplementary and complementary angles.

demonstrate an understanding of the properties of quadrilaterals.

identify the legs and the hypotenuse of a right triangle.

accurately draw triangles using the SSS, SAS, or ASA conditions.

solve problems using percents.

determine the correct unit of measure to use in a real-life application.

translate words to algebraic expressions or equations.

create and apply an algebraic equation to solve a real life situation.

justify responses to problems.

find the next term, the constant difference, and a rule for an arithmetic sequence.

generalize a solution to solve a new problem.

demonstrate an understanding of size change and its affect on the perimeter and the area of a geometric figure.

make conclusions from a stem-and-leaf plot.

make conclusions from a histograph.

identify the range, mean, median, and mode of a data set.

create a box-and-whisker plot from a data set.

demonstrate an understanding of simple probability.

make conclusions from a Venn diagram.

fit a line to data.

explore and recognize various key sequences on the scientific calculator.

use a graphics calculator to visualize an equation.

find the appropriate window on a graphics calculator to visualize an equation.

explore some functions and editing features on a graphics calculator.

use the perimeter, area, surface area, and volume formulas to evaluate problems involving various geometric figures.

use the circumference and area formulas to evaluate problems involving circles.

solve algebraic equations.

find the third side of a right triangle using the Pythagorean Theorem.

demonstrate an understanding of absolute value.

multiply, divide, and simplify rational numbers using the power rules.

Prealgebra

Instructions for the User

- **To use Summer Math Skills Sharpener, simply tear off a page and complete it. The program is designed to be used 3 days per week for 10 weeks.**
- **Supplemental lessons have been included to address changing state and National Standards. These lessons are OPTIONAL. (See green supplement following body of program.)**
- **Detailed solutions to all the problems are included at the back of the book. Please complete an entire sheet prior to checking your answers. You may want to remove the solutions and store them in a place apart from the book.**
- **All concepts are part of a standard prealgebra curriculum. Please attempt all problems. In addition to the solutions, pink "Help Pages" have been included to assist you in completing the problems.**
- **A yellow "Glossary of Terms" is located at the back of the book.**
- **Pages should be worked in order. While each page contains mixed concepts, individual concepts, within the book, have been ordered from easier to more difficult.**
- **If you experience difficulty with certain concepts, address the problem with your teacher. He or she may recommend additional help in these areas.**
- **It is important to give every problem your best effort. Problems may seem challenging, but use a combination of the "Help Pages" and the "Solutions" to assist you for maximum success.**
- **We appreciate your comments. Please complete the enclosed evaluation page after you have entered your next math course but before November 1st.**

For problems 1 – 4, simplify.

1. $|-5|$ 2. $-|-5|$ 3. -5^2 4. $(-5)^2$

5. **Convert each of the following:**

   a. 32 in. = _____ ft. b. 48 oz. = _____ lb.

   c. 10 ft. = _____ yd. d. 6 qt. = _____ gal.

6. **The wavelength of x-rays, in centimeters, is 3.048×10^{-9}. Rename this number in decimal notation.** _____

7. **28 is what percent of 50?**

   a. **Write the number sentence (equation) to represent this situation.**

   b. **Solve this equation.**

8. **Everything in a store is discounted 10%. The original price of an iPod is $249.99.**

   a. **Find the new price.** _____

   b. **If sales tax is 6%, then what will be the final cost?** _____

9. **Calculate each quotient. Use a bar to show repeating decimals.**

   a. $\frac{2}{3}$ _____ b. $\frac{1}{6}$ _____ c. $\frac{5}{6}$ _____

   d. $\frac{1}{9}$ _____ e. $\frac{1}{7}$ _____ f. $\frac{1}{11}$ _____

10. What number am I?

a. The sum of my digits is 9.

b. I am odd.

c. I am more than 100.

d. I am a power of 3.

11. Write in exponential form.

a. $4 \times 4 \times 4 \times 4 \times 4$ **b.** $2.1 \times 2.1 \times 2.1$ **c.** $y \times y \times y \times y \times y \times y \times y$

_____ _____ _____

12. Below is a graph that shows the ages of the American Presidents at the time of their inauguration.

42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69

a. What is the range of the ages? _____

b. What is the median age? _____

c. Theodore Roosevelt was the youngest President ever elected. How old was he at the time of his inauguration? _____

d. How many Presidents were 50 or younger at the time of their inaugurations? _____

For problems 1 – 4, solve for x.

1. $2x-1=5$ **2.** $\frac{2}{3}x+1=7$ **3.** $-x+5=2$ **4.** $\frac{3}{5}x=\frac{9}{10}$

5. Write the following sentence in algebraic notation. ***The output is the product of six and the sum of five and a number.*** _____

6. Simplify each of the following expressions:

a. $7a+9a$ **b.** $8x-10x$ **c.** $6ab+3ba$

7.

**	**	**	**
	**	**	**
		**	**
			**

a. Draw figure 5.

b. Each consecutive (next) figure has _____ more asterisks than the previous figure.

c. Predict the number of asterisks in figure 10. _____

8. a. Complete the table below.

x	0	1	___	___	4	5
y	10	7	4	1	___	___

b. Find the slope between any two points.

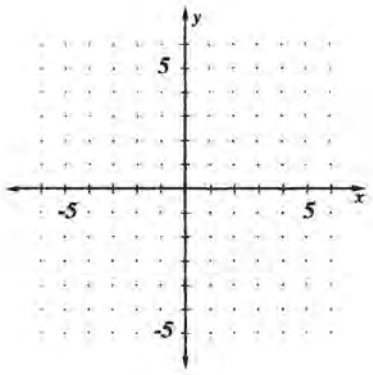
c. Create a rule to find any y value in terms of x. _____

d. Find y when $x=10$. _____

9. If $x = -4$, find $-2x^2 - x - 4$.

10. a. Find three pairs of numbers that satisfy the equation: $2x + y = 5$

b. Plot the points on the graph at the right and draw a line through them.



c. For every x change of one, y changes by _____. This is another description of slope.

11. FOUR 4's. Using only four 4's, the operations (+, −, ×, ÷), and parentheses, write the expressions to equal the values of the digits. (The digits are the numbers 0-9.) For example $4 \div 4 + 4 \div 4 = 2$

12. (Optional calculator) Evaluate.

$$\frac{-5 + \sqrt{5^2 - 4(1)(4)}}{(2)(1)}$$

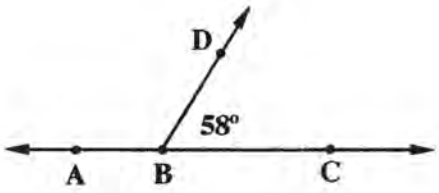
For problems 1 – 3, solve for x.

1. $\frac{1}{2}x - 5 = 10$

2. $2x - \frac{1}{4} = -\frac{1}{2}$

3. $\frac{2}{3}x - 8 = 4$

4. **Two angles are *supplementary* if their sum measures 180°. Find $m\angle ABD$.**



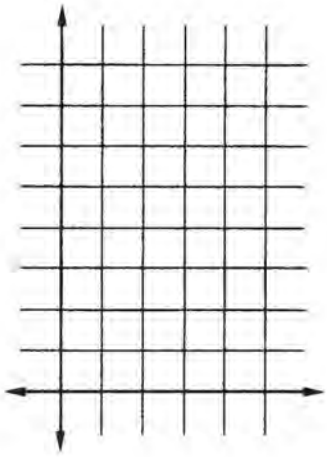
5. **Kristen baby sits for $5.25 per hour.**

   a. **Complete the table to show her wage (w) for the hours (h) worked.**

h	1	2	3	4	5	h
w	5.25					

   b. **Write an equation for her wage (w) in terms of hours (h).**

   c. **Graph this equation on the coordinates at the right.**



   d. **What number represents the constant rate of change?** _____

6. **16 is 12% of what number?**

7. **Find two numbers whose product is 42 and whose difference is 1.**

8. Use <, >, or = to make each comparison true.

a. 1 liter _____ 1 quart **b. 1 kilometer _____ 1 mile**

c. 1 gram _____ 1 ounce **d. 1 kilogram _____ 1 pound**

9. Consider the expression $4w + 3x^2 - 7y - 2w$.

a. How many terms are in this expression? _____

b. What is the coefficient of y? _____

c. Name two *like* terms. _____

d. What is the degree of $3x^2$. _____

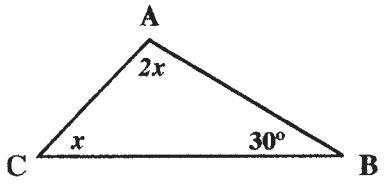
e. Simplify this expression. _____

10. If $x^n = 125$ and x and n are integers between 1 and 10, find x and n.

$x =$ _____ **$n =$ _____**

11. The sum of the angles of a triangle is 180°.

a. Write a number sentence to represent the situation pictured at the right.



b. Find $m\angle C$ and $m\angle A$.

12. The number 3^4 means _____ is a factor _____ times.

1. **The commutative and associative properties for real numbers can be used to simplify problems. Solve the following problems mentally by rearranging (commutative property) and regrouping (associative property) the numbers.**

   a. **$2.25 + $2.98 + $4.75 =**

   b. **$-2 + 6 + 2 + 9 =$**

   c. **$-4.5 + 2.9 + -5.5 + 3.1 =$**

   d. **$\frac{1}{2} + \frac{3}{4} + \frac{1}{2} + \frac{1}{4} =$**

2. a. **b^3 means _____ is a factor _____ times.**

   b. **b^5 means _____ is a factor _____ times.**

   c. **$b^3 \bullet b^5$ means _____ is a factor _____ times. Therefore, $b^3 \bullet b^5 = b$ ☐.**

   d. **Generalize: $b^m \bullet b^n = b$ ☐.**

3. **Cooper drove from Chicago to St. Louis, a distance of 300 miles. The trip took 6 hours to complete.**

   a. **Find his average rate of speed in miles per hour.**

   b. **Convert his rate to feet per second.**

4. **A middle school intramural bowling team had the following scores for its first outing:**

   25 55 70 88 95 101 115 130 150 168 172

   a. **What are the upper and lower extremes? _____**

   b. **What is the range of scores? _____**

   c. **What is the median score? _____**

   d. **What is the lower quartile? _____**

   e. **What is the upper quartile? _____**

   f. **Represent this data using a box-and-whisker plot.**

20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180

5. a. Accurately draw ΔXYZ where $XY = 2in$, $YZ = 1in$, $m\angle Y = 55°$.

b. Explain why all triangles drawn with these dimensions will be congruent.

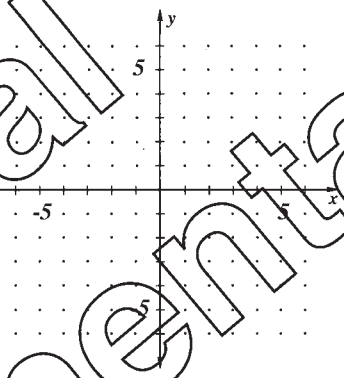
6. Given: ΔABC, $A = (-3, 3)$, $B = (-5, 1)$, $C = (-3, 1)$

a. $T_{3,-2}(\Delta ABC) =$

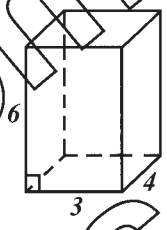
b. $r_{x-axis}(\Delta ABC) =$

c. $r_{y-axis}(\Delta ABC) =$

d. Draw the image of each on the coordinates at the right.



7. a. Draw a net for the figure pictured at the right.



b. Find the surface area.

8. a. In a plane, the intersection of two nonparallel lines is a _____.

b. In space, the intersection of two nonparallel planes is a _____.

c. In space, the intersection of three nonparallel planes is a _____.

d. Give a real world example of intersecting lines.

e. Give a real world example of intersecting planes.

HUNDRED CHART

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

absolute value : The distance a number is from zero on a number line. Distance is always positive.
ex. $|2| = 2$, $|-2| = 2$

acute angle : an angle measuring greater than 0 degrees and less than 90 degrees.

algebraic equation : a math sentence relating two expressions as equal.

algebraic expression : a combination of numbers and variables joined by the operations of arithmetic.

altitude : the perpendicular distance from the vertex of a triangle to the side opposite. Also, the perpendicular distance between parallel lines.

angle : the union of two rays (the sides) at a point (the vertex).

area : the number of unit squares or parts of unit squares required to tile a two dimensional figure.
ex. parallelogram : $A = hb$ trapezoid : $A = \frac{1}{2}h(b_1 + b_2)$ triangle : $A = \frac{1}{2}hb$
rectangle : $A = hb$ circle : $A = \pi r^2$ square : $A = s^2$

arithmetic sequence : sequence with a constant difference.

base : in the expression b^n the variable b is the base.

binomial : a polynomial containing two terms.

circle : the set of all points (the radius), equidistant from a certain point (the center).

circumference : the perimeter of a circle. The ratio of the circumference to the diameter is π.
$c = \pi d$ gives the circumference for a circle with diameter d.

coeffiecient : the number in a term.
ex. $-2x$, the -2 is the coefficient.

complementary angles : two angles that have a sum of 90°.

composite : a number that has more than two factors.

ORDER OF OPERATIONS : Parentheses, powers, multiplication, division, addition, subtraction.

(use "Pretty Please, My Dear, Aunt Sally" to help remember)

ex. $(2 - 3) \times 4 - 8 \div 2^2$

1. Parentheses first : $-1 \times 4 - 8 \div 2^2$

2. Powers second : $-1 \times 4 - 8 \div 4$

3. Multiplication and/or Division third : $-4 - 2$

4. Addition and/or Subtraction of LIKE terms last : -6

ALGEBRA LANGUAGE :

VARIABLE : A variable is a symbol that can be replaced by a number.

ex. $x + 8 = 13$. **The** x **is a variable. In this case,** $x = 5$ (x **is replaced by 5) makes this a true sentence.**

A variable also can be a part of an expression.

ex. $4m$. **The** m **is the variable. The 4 is the coefficient of** m.

ALGEBRAIC EXPRESSION : When numbers and variables are joined by the operations of arithmetic an algebraic expression is formed.

ex. Two less than x **is written as** $x - 2$.

Three times a **is written as** $3a$.

Notice that any substitution of a number for the variable in an expression is true.

ALGEBRAIC SENTENCE : An algebraic sentence occurs when algebraic expressions are joined by a math verb. Math verbs are $<, >, \leq, \geq, =, \neq,$ **and** $\approx$. **Think of writing a math sentence as translating from English language to math language.**

ex. Twice a number is 8. $2x = 8$

Four more than a number is three times its opposite. $4 + n = 3(-n)$ **or** $4 + n = -3n$

Three times a number is greater than 20. $3n > 20$

EQUATION : When two expressions are equal, the math sentence is an equation.

ex. $3x - 2 = 10$

FORMULA : A formula is an equation that states that a single variable equals an expression.

ex. $A = bh$

Lesson 1 pg. 1

① $-8+8=0$

② $8+2(3)-8$
$8+6-8$
$14-8=6$

③ $12\div4\times2$
$3\times2=6$

④ $\frac{4}{6}=\frac{2}{3}$

⑤ $\frac{4}{9}$ Already simplified.

⑥ $\frac{15}{25}=\frac{3}{5}$

⑦ $\frac{4}{12}=\frac{1}{3}$

⑧ a. $\frac{1}{4}=.25$ b. $\frac{3}{4}=.75$ c. $\frac{1}{2}=.5$
d. $\frac{3}{2}=1.5$ e. $\frac{1}{8}=.125$ f. $\frac{3}{8}=.375$

⑨ a. 2 b. 5

⑩ 9.3025 smallest
9.38 ↓
9.4219
9.44 largest

Lesson 1 continued pg. 2

⑪ 107 students ÷ 18 suckers / bag. Consider 100 students ÷ 20 suckers / bag = 5 bags, but there are more than 100 students and less than 20 suckers / bag, so your will need 6 bags.

⑫ 1 inch = 25 miles, so 3 inches = 75 miles
and $\frac{1}{2}$ inch = 12.5 miles
therefore $3\frac{1}{2}$ inches = 87.5 miles

⑬ 3 pencils = \$1.29. Since you only have \$10.00, you must determine how many \$1.29's are in \$10.00. $7\times\$1.29 = 7\times1+7\times.29 = 7+2.03 = \$9.03$. This means you can purchase 21 pencils. This assumes that the pencils come in a 3-pack.

⑭

HUNDRED CHART

f. The least common multiple of 2, 3, and 5 is 30. 30 is the first number in the chart that contains all three markings for multiples of 2, 3, and 5.

g. The greatest common factor of 27 and 36 is 9. 9 is the largest number in the chart that contains the marking that is common to 27 and 36, which is a multiple of 3.

Lesson 2 pg. 3

① $4+-8\times2$
$4+-16=-12$

② $5-8\div2\times3$
$5-4\times3$
$5-12=-7$

③ $24\div4\times6$
$6\times6=36$

④ a. 6 b. 7 c. 490,000.00000 d. 486,195.2679

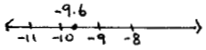
⑤ a. The first three numbers that contain the markings for multiples of 2 and 3 are 6, 12, and 18.
b. 2, 3, and 7
c. 15 and 24 share a common multiple of 3.
d. 2, 3, 5, 7, 11, 13

⑥ $\frac{2}{3}\cdot\frac{2}{2}=\frac{4}{6}$ $\frac{2}{3}\cdot\frac{3}{3}=\frac{6}{9}$ $\frac{2}{3}\cdot\frac{4}{4}=\frac{8}{12}$

Lesson 2 continued pg. 4

⑦ a. $\frac{1}{10}=.1$ b. $\frac{3}{10}=.3$ c. $\frac{2}{5}=.4$
d. $\frac{1}{100}=.01$ e. $\frac{25}{100}=.25$ f. $\frac{9}{10}=.9$

⑧ a. $ab=8\times-2=-16$ b. $a+b=8+-2=6$
c. $a-b=8-(-2)=8+2=10$ d. $a\div b=8\div-2=-4$
e. $\frac{1}{2}a=\frac{1}{2}(8)=4$ f. $(-2)^2=4$

⑨ a. 

-10 and -9
b. -9.6 is closest to -10

⑩ a. $8.1\times5.2\approx8\times5=40$
b. $2.9\times3.1\approx3\times3=9$
c. $20.1\times7.9\approx20\times8=160$
d. $9999\times5.1\approx10000\times5=50000$

⑪ 15, 19

⑫ p = profit
c = \$158
s = \$237
$p=s-c$ or
$p=237-158=\$79$

